The Monopolist's Market with Discrete Choices and Network Externality Revisited: Small-Worlds, Phase Transition and Avalanches in an ACE Framework

Denis Phan, Stéphane Pajot[1] and Jean-Pierre Nadal[2]

[1] ENST de Bretagne and ICI Université de Bretagne Occidentale (denis.phan@enst-bretagne.fr, http://www-eco.enst-bretagne.fr/∼phan)
[2] Laboratoire de Physique Statistique, Ecole Normale Supérieure, 24 rue Lhomond, 75231 Paris cedex 05, France (nadal@lps.ens.fr, http://www.lps.ens.fr/∼nadal)

Abstract. In this paper, we explore the effects of the introduction of localised externalities through interaction structures upon local and global properties of the simplest market model: the discrete choice model with a single homogeneous product and a single seller (the monopoly case). Following Kirman, the resulting market is viewed as a complex interactive system with a communication network between entities. We use an ACE (Agent based Computational Economics) approach to investigate corresponding market mechanisms and underline in what way the knowledge of generic properties of complex adaptive system dynamics can enhance our perception of the market mechanism in the numerous cases where individual decisions are inter-related. More specifically, we discuss analogies between simulated market mechanisms and classical phenomena in the physics of disordered systems such as phase transition, symmetry breaking, avalanches and long range dependence. Various network structures are taken into account: as regular network (lattices) and random networks represent two limiting cases of localised interaction structures, the so-called "small-world" networks are an intermediate form between these two extremes. The first and second sections are devoted to the local and global dimension of the related dynamics, while the third section is dedicated to first investigations into the incidence of externalties and network structure upon the optimal asymptotic price for a monopolist.

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1 Introduction

In this paper, we explore the effects of the introduction of interaction structures (structured externalities) upon local and global properties of the simplest market model: the discrete choice model (Anderson *et al.* [2]) with a single homogeneous product and a single seller (the monopoly case).

Following Kirman [55–57] such a market is viewed as a *complex interactive system* with a communication network. From this perspective, ACE (Agent based Computational Economics - [94,95,91,110]) is a promising approach to investigate market mechanisms (cf. for instance among others: Vriend [100]; Kirman and Vriend [61]; Wilhite [108]). That is, following Mirowski and Somefun [73], markets will be viewed "*as evolving computational entities*". In such a system, buyers as well as sellers may be represented by a suitable software agent. Each agent is then linked by communications structures to other entities of the system. In our model, the market embodies communications between customers and seller, and the neighbourhood embodies communications customers. In this way, such an agent may exchange information with his environment, to adapt his behaviour given this information (individual learning). As a consequence, each agent thus contributes to the adaptation of the whole system (collective learning, following Dosi *et al.*, [31]). The resulting system may be viewed as a *complex adaptive system* [105,90] involving social dynamics (Durlauf and Young, [37]). In this paper, we underline in what way the knowledge of generic properties of complex adaptive system dynamics can enhance our perception of the market mechanism in the numerous cases where individual decisions are inter-related. Simulations are supported by the Moduleco ACE Lab [83–85,112].

1.1 Motivation: the role of individual interdependence in market behaviour

While the market is given as the main object of economic science, few authors have really discussed the definition and ontology of the market, as underlined by Hodgson [48] or Auerbach [7]. The market appears to be both the central institution for coordination within capitalism and also an organisation which can take many different forms [71]. Nevertheless, numerous formal works concentrate upon questions such as the existence, stability and efficiency of the equilibrium, neglecting both the processing and the informational dimension of the market underlined by Hayek [47]. As a result, informational, cognitive, organisational and institutional dimensions of the market are eliminated from this traditional perspective. In order to overcome some of these limitations Kirman [57–59]) points out, in particular, the role of networks in market organisations. This theoretical point of view, putting together Networks and Markets, joins other approachs followed by some sociologists showing interest in interdisciplinary cooperation [87,39].

That is, economic sociology puts the emphasis on the diversity of organisational forms of markets, as well as on the role of network relationships. More specifically, the structural method [65] is devoted to the study of the relationship between a stylised form of the social system of relationship (the structure) and the behaviour of the members of such a system. Various authors put the emphasis on different structures of networks like, for instance, Burt ([23]: "structural hole") or Granovetter ([43]: "weak ties"). For the later, the effects of group membership on economic decisions, are marked with the concept of " embeddedness" . The approach suggested by Granovetter [45,46] shows that economic action is based on the networks of relations between people. In other words, economic action is localised socially and cannot be explained by merely individual causes.

1.2 Dynamic behaviour

More specifically, following [36,86] we discuss analogies between simulated market mechanisms and classical collective phenomena studied in statistical physics, especially in the case of disordered (heterogeneous) systems. The macroscopic properties of a physical system composed of a large number of interacting units (atoms, molecules, etc.) can be summarised in a *phase diagram* in the space of the control parameters (e.g. temperature, external field, etc.). This space can be decomposed into domains, each domain corresponding to a *phase* (e.g. liquid or solid, ferromagnetic or paramagnetic, etc.). The *phase transitions*, occurring at points or lines separating these domains, correspond to the emergence of a different order, associated with the restoring or the breaking of a symmetry (see e.g. Anderson and Stein, [1]). For instance, a ferromagnetic state will appear below some critical temperature Tc, with the breaking of the symmetry " up/down" in a system of magnetic moments ("spins"). The phase transitions themselves can be classified in categories, each class corresponding to a same qualitative set of properties. In the case of a "first order transition", a jump occurs in the macroscopic quantity which characterises the new order, and the transition occurs at different values of the control parameter, depending on whether the latter is continuously increased or lowered (hysteresis effect). Moreover heterogeneity may produce *avalanche* phenomena and more complex hysteresis effects (see e.g. Sethna *et al.*, [92]). Such behaviour will be described more precisely in section I, for the microscopic (local) dynamics, and section II, for the resulting macroscopic dynamics.

According to physics' results, when individual actions are made to be interdependent, complex dynamics may arise. That is the case, for instance, when agents locally interact over a specific network. In such cases, Axtell [9] has underlined the effects of distinct agent interaction structures in multi-agent models. In this paper, we review the effect of various network structures. Regular network (lattices) and random networks represent two limit cases, the so-called "small-world" networks [77,101–103] being an intermediate form between these two extremes.

The first and second sections are devoted to the local and global dimension of the market dynamics, while the third section is dedicated to first investigations into the incidence of externalties and network structure upon the optimal asymptotic price for a monopolist.

2 Local Behaviour

The question of social influence over individual choice, or, in other words, the trade-off between "*Individual Strategy and Social Structure*" (Young, [109]) is now on the economist's agenda [6,37,62]. This question has been addressed for a long time in other fields of social science (Schelling [88,89]; Granovetter, [45]; see also: Kindermann and Snell [53,54]; Weidlich and Haag, [104], for formal aspects). In order to introduce market dynamics linked with both individual idiosyncratic preference and network social influence, we will first borrow from social science the notions of individual and collective thresholds. After a short overview of economists' formal contributions to this question, we propose two typologies: a typology of the interactions between individual choices and demand dynamics, and the Watts and Strogatz [103] typology of network structures. After introducing a family of formal models of individual choice, we qualitatively explain the local dynamics and the avalanche phenomenon with an example.

2.1 Individual choice and social structure

In agents' models with an individual threshold such as in Schelling [89] or Granovetter [44], the individual threshold of adoption gives a definition of the number of adopters each agent considers sufficient to modify his behaviour. The equilibrium is fully determined from the knowledge of the distribution of individual thresholds. In the example of a riot [44], each agent has an individual threshold which corresponds to the number of people in the riot that he considers necessary for him to choose to join it. For a population of 100 people, the distribution of thresholds is uniform (an agent with threshold 0, an agent with threshold 1, an agent with threshold 2, and so on, the last agent having threshold 99). The individual with threshold 0 is the instigator of the dynamic. He decides to adopt deviant behaviour and, for example, breaks a window. Consequently, the agent with threshold 1 joins him and he acts in an illegal way, which influences another agent and so on (*chain reaction*). Gradually, the riot grows and reaches the equilibrium with all the population affected. This mechanism is called a "domino effect" or a "bandwagon effect". It corresponds to what is called an "*avalanche*" in the physics literature. Here, because the distribution is uniform, the avalanche size is equal to the total population, as we will see below. If the threshold distribution is modified, the equilibrium changes. Let us suppose that the individual with threshold 1 is replaced by an individual with threshold 2, so the dynamic is limited to one person. The revision of the situation of only one person in the population leads to a profond modification of the global behaviour. There is no chain reaction and thus no avalanche.

In the context of these models, there is no "local network" in the sense that individuals are only sensitive to the percentage of the total population which has previously adopted (a behaviour, product, etc.). The neighbourhood of each agent is composed of the set of all other agents. These are called global interactions. Valente [99] stresses the importance of the structures of interpersonal relations in the propagation phenomenon (innovations, opinions, goods) by distinguishing for each agent, the " collective behaviours threshold" and the " threshold of exposure" . The *threshold of collective behaviour* corresponds to the threshold such as it is defined in the models of *global interactions*: the number of previous adopters in the total population before the agents adopt it themselves. By opposition, the threshold of exposure is based on the *localized interactions* (i.e. the interpersonal relations between agents, including both *local* (short distance) interactions and long distance interpersonal interactions). The *threshold of exposure* of an individual is defined by the proportion of adopters in his personal network (neighbourhood) when he changes his behaviour. Taking into account the network of individual relations is essential because it makes it possible to distinguish the agents who are conservative or resistant (high threshold of exposure) and those who are innovators (small threshold of exposure) and exposed only tardily to the product.

Such considerations can be completed in a more formal way. In the mathematical sociology field, Weidlich and Haag [104] propose, in the global perspective, a generic model of opinion formation based upon a *master equation* and *Fokker-Plank* approximation approach. Kindermann and Snell [54] identify a social network as an application of a *Markov random field*. Galam *et al.* [42] propose another application of statistical mechanics tolls to sociology in a work qualified as "sociophysics". In economics, the pioneering work of Fölmer [40] applies local stochastic interactions by the way of Markov random fields in a general equilibrium model with uncertainty.

In the 90's, new stochastic models were introduced in order to take into account informational phenomena in financial markets, such as informational cascades [14] and correlative bubbles. Similarly, the master equation and the Fokker-Plank approximation are used by Topol [96] and Orléan [79–81]. Such methods in the social sciences were inspired by Weidlich and Haag [104] but also by Aoki [3].

At the local level, the pioneering work of Ellison [38] in the field of evolutionary games theory, followed by the contributions of Blume [15,16] and the early works of Durlauf on interdependent growth phenomena [32–34] again introduce a statistical mechanism tool as a way to take into account interaction phenomena in economics (see syntheses by [35,21,17,50,86] among others); for a discussion of the relationship with mechanical physics, see [36,86]. More generally Young [109] proposed an essay on *Individual Strategy and Social Structure*, and the collective works edited by Durlauf and Young [37] provide useful new developments.

In Europe, similar programs emerged, e.g. in [68,24,62,20] among others. The example of the analysis of the fish market in Marseille [60,58] opens some other opportunities to make use of methods and concepts taken from statistical mechanics (see for instance [75,106]) as well as ACE (Kirman and Vriend, [61]).

Finally, beyond the papers cited, social influence as well, like in the ACE approach, is widely used to model financial markets (for a survey, see: LeBaron, [66,67]).In recent years, a growing field of so-called "econophysics" has developed a complementary approach within the physicists' community (see for instance [111,70]). Within the important part of this literature devoted to mathematical finance, several models address the effect of interactions between agents [70,26,25] on the market.

In the present paper we focus on the effect on market behaviour of interdependencies between customers, and we thus consider a single seller - the case of a monopoly market. We propose and study an economic model which allows us to highlight the economic effects of the structure on the global level of demand. Three situations are possible (Table 1).

Table 1. A typology of interactions and demand dynamics

Neighbourhood *(Moduleco*)*	(a) No relations *(empty)*	(b) Localised relations *(neighb2, neighb4,...)*	(c) Generalised relations *(world)*
Level of interactions	(independent agents)	Localized interactions	Global interactions
Demand sensitivity to the network topology	Null	Strong	Null
Avalanches	No	Possible (localised in the network)	Possible -not localised in the network)

* on neighbourhoods in Moduleco, see Table 2 below

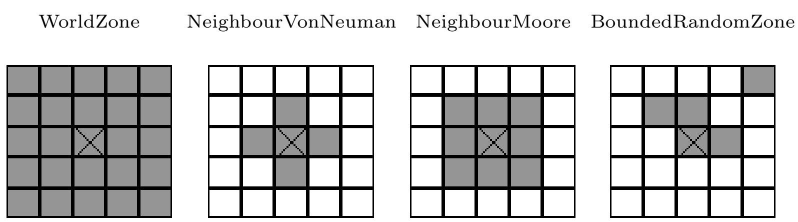
In the first extreme case (a), there are no relations between agents (Moduleco: "empty" neighbourhood). In this case, the aggregate demand is independent of the structure and no external effect (local or global) is present. The agents are independent one from the other.

In the second extreme case (c), all agents are connected by direct relations in the system (Moduleco: "world" neighbourhood). All agents are equivalent in the network and they interact by means of global interactions. In this way, the aggregate demand is sensitive to the global external effect initiated by the sensitivity of agents to the choices of others, but remains independent of the topology of the network (because the neighbourhood of each agent is composed of all the other agents). Thus, avalanches are not localised on the network but appear in a dispersed way within the system.

Finally, the intermediate case (b) corresponds to situations where agents have specified relations (regular neighbourhood or not). The agents are not all directly connected, one to the other. Interactions are local and the topology of the interpersonal network influences the aggregated demand on the market. This local interdependence gives rise to localised avalanches on the network.

Table 2. Neighbourhood in Moduleco [112]

In Moduleco, all relationships between agents are supported by specific Mediums. Such classes define how agents interact and how they are connected together. For example, NeighbourMedium allows Moduleco to define the set of neighbours an agent can have. Once his neighbourhood defined, an agent can invoke the services of his neighbours, such as getting specific information, for instance. Neighbours have specific subclasses for each specific topology such as WorldZone (all agents in the grid), NeighbourVonNeuman (North, South, East and West agents of the current agent on a grid) and Neighbour8 (the 8 closest agents on a circle). As a result, the communication topology is defined by the Neighbourhood. The grid is just an easy way to represent agents on a screen (that is offered by default, but that can be changed, as usual). For heuristic purposes, a circle representation is available, useful for the one-dimensional, periodic lattice.



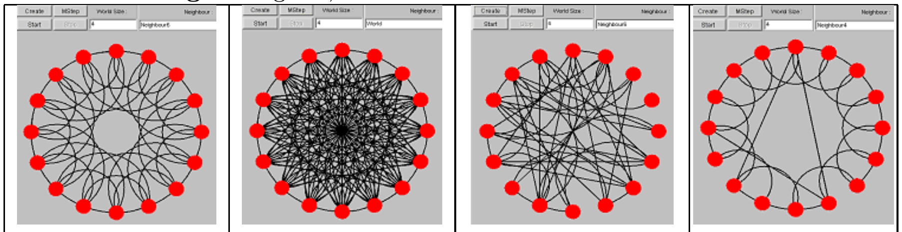
A random neighbourhood is also available like with, for instance, a BoundedRandomZone topology. A dynamic neighbourhood is also available, for instance with random pair-wise coupling at each step or neural network activation of virtual links. Finally, it is possible to perturb a regular network by rewiring some links, in the way of the so-called "small-worlds"

2.2 Network structure: regular, random and "small-world"

Following an important body of literature in the field of socio-psychology and sociometrics, initiated by Milgram [72], the "six degrees of separation" paradigm of a "small-world", Watts and Strogatz [103] proposed a formalisation in the field of disordered systems. The original Watts and Strogatz (WS) small-world starts from a regular network where n agents are on a circle (one-dimensional, periodic lattice) and each agent is linked with his $2k$ nearest neighbours.

In the WS rewiring algorithm, links can be broken and randomly rewired with a probability p. In this way, the mean connectivity remains constant,

Fig. 1. Regular, random and "small-world" networks



(a) Regular network connectivity $k = 6$	(b) Regular network full connectivity	(c) Random network connectivity constrained	(d) Small-world $k = 4$; 3 links rewired

but the dispersion of the existing connectivity increases. For $p = 0$ we have a regular network and for $p = 1$ a random network. Intermediate values between 0 and 1 correspond to the mixed case, where a lower p corresponds to a more local neighbour-dependent network. The version of the algorithm implemented in Moduleco took h nodes, broke i links for each of these nodes and randomly rewired the broken links with other nodes. The parameter $q = \frac{hi}{n}$ here plays a role analogous to the one of p.

A large range of small-world properties is now well known [101,102,77]. Barthelemy *et al.* [12] provide a typology of small-worlds, with related properties, including both Watts-Strogatz and some varieties of "scale free" topologies [10,11].

Following Watts [101], Barthelemy *et al.* [12] and Holme [49], two main structural indicators characterise a network through both the local and global dimensions of its connectivity. These indicators use the language of graph theory [13,18]. Accordingly, each node (agent) is called a "vertex" and each link an "edge". The connectivity of a vertex is the number of edges attached to the vertex.

The first indicator is the *clustering coefficient* C, defined as the average ratio of the number of existing edges between neighbours of a vertex to the maximum number of possible edges. In a fully connected network $C = 1$; in a random network $C \simeq< k > /n$. In contrast, small-world networks have values of C of the same order of magnitude as those of regular lattices.

The second important indicator is the *characteristic path length* L. This is the average value of the shortest path length $d(i, j)$ between all possible pairs of vertices (i, j). For a random network and a small-world (with $0.001 < p < 0.01$), the value of L behaves as $L \sim \ln(n)/\ln(k)$, while for a regular lattice $L \sim n/(2k)$, where n is the number of vertices and k is the average connectivity of the network.

Watts et Strogatz [103] have underlined that many real networks have a small characteristic path length as in the "six degrees of separation" of Milgram, and a high clustering coefficient (Table 3). In the *Kevin Bacon Graph*, vertices are actors in IMDb (*http://www.imdb.com*); an edge between two actors means that both have acted in a specific movie. In the *Western States Power Grid*, edges are high-voltage power lines and vertices are transformers,

Table 3. "Small-world" in the real world: social, technical and biological networks

	Kevin Bacon	W.S.Power Grid	C.Elegans Graph
	225 226	4941	282
k average connectivity	61	267	14
L characteristic path length	3,65	18,7	2,65
C clustering coefficient	0,79 0,02	0,08	0,28

Source: [49] http://www.tp.umu.se/∼holme/seminars/swn.pdf

generators, etc. The *C. Elegans Graph* describes the neural network of the Caenorhabditis Elegans worm, with nerves as edges and synapses as vertices [49].

In economics, the small-world architecture has been applied by Jonard to bilateral games [51,52], and to knowledge and innovation diffusion processes (with Cowan *et al.* [28,29]). Market models have been developed by Wilhite [108], among others.

2.3 Modelling the individual choice in a social context

This model deals with the simplest discrete choice problem (Anderson *et al.*, [2]): binary choice. Analytical results presented below follow a companion paper by Nadal *et al.* [74] [1]. The present paper focuses more specifically on dynamic aspects, including effects depending on the network architecture.

We consider a set Ω_N of N agents with a classical linear willingness-to-pay function. Each agent $i \in \Omega_N$ either buys ($\omega_i = 1$) or does not buy ($\omega_i = 0$) one unit of the single given good of the market. A rational agent chooses ω_i in order to maximize his *surplus function* V_i:

$$\max_{\omega_i \in \{0,1\}} V_i = \max_{\omega_i \in \{0,1\}} \omega_i (H_i + \sum_{k \in \vartheta_i} J_{ik}\omega_k - P), \tag{1}$$

where P is the price of one unit and H_i represents the idiosyncratic preference component. Some other agents k, within a subset $\vartheta_i \subset \Omega_N$, such that $k \in \vartheta_i$, hereafter called neighbours of i, influence agent i's preferences through their own choices ω_k. This social influence is represented here by a weighted sum of these choices. Let us denote J_{ik} the corresponding weight i.e. the marginal social influence on agent i, of the decision of agent $k \in \vartheta_i$. When this social influence is assumed to be positive ($J_{ik} > 0$), it is possible, following Durlauf [35,36], to identify this external effect as a *strategic complementarity* in agents' choices [22,27].

Formally, if we define the social influence component by a continuous $\mathcal{C}^2$ function: $S(\omega_i, \omega_{-i})$ where ω_{-i} is the vector of the neighbours' choices :

$$S(\omega_i, \omega_{-i}) \equiv \omega_i \sum_{k \in \vartheta_i} J_{ik}\omega_k \tag{2}$$

[1] available at: http://www-eco.enst-bretagne.fr/∼phan/papers/npgweiha2003.pdf

the social influence component in specification (1) appears to be a restriction of (2) at binary arguments $\{0,1\}$. In the continuous case, the marginal social parameter J_{ik} appears to be the second order cross-derivative of $S(\omega_i,\omega_{-i})$ with respect to ω_i and ω_k, according to the definition of strategic complementarity:

$$\frac{\partial^2 S(\omega_i,\omega_{-i})}{\partial\omega_i\partial\omega_k} = J_{ik} > 0 \tag{3}$$

For simplicity, we consider here only the case of *homogeneous* influences, that is, identical positive weights $J_{ik} = J_{\vartheta_i}$ for all influence parameters in the neighbourhood of i. That is, if N_{ϑ_i} denotes the number of neighbours of agent i, we have :

$$J_{ik} = J_{\vartheta_i} \equiv J/N_{\vartheta_i} > 0 \qquad \forall i,k \in \Omega_N \tag{4}$$

For a given neighbour k taken in the neighbourhood ($k \in \vartheta$), the social influence is J_{ϑ_i} if the neighbour is a customer ($\omega_k = 1$), and zero otherwise. Individual influence is inversely proportional to the size of the neighbourhood. As the cumulated social effect is the sum of individual effects over the neighbourhood, social influence depends on the proportion of customers in the neighbourhood. In a *regular* network (N_{ϑ_i} constant and equal to N_ϑ for all $i \in \Omega_N$), all individual effects have the same magnitude over the network (equal to: $J_\vartheta \equiv J/N_\vartheta$). Conversely, in a small-world network or in a random network, the magnitude is inversely proportional to the size N_{ϑ_i} of the given neighbourhood.

Depending on the nature of the idiosyncratic term H_i, the discrete choice model (1) may represent two quite different situations [2]. In this paper, each agent is assumed to has a willingness to pay that is *invariable* in time, but may differ from one agent to the other. As consequence, private idiosyncratic terms H_i are randomly distributed among the agents at the beginning, but remain fixed during the period under consideration. It is useful to introduce the following notation:

$$H_i = H + \theta_i, \tag{5}$$

[2] Following the typology proposed by Anderson *et al.* [2]), we distinguish a "psychological" and an "economic" approach to individual choice. Within the psychological perspective (Thurstone (1927- the TP case -), the utility has a *stochastic* aspect because: *"there are some qualitative fluctuations from one occasion to the next ... for a given stimulus"* [97]). In this paper, our approach is closer to the McFadden [69] one, in which each agent have an idiosyncratic willingness to pay that is *invariable* in time, but non observable by the seller. In a "risky" situation the seller knows the statistical distribution of this characteristic over the population before social influence (McF case). However, stochastic utility is closer to the BBD generic model of interaction [35,17,21]. For a comparison between the TP case and the McF case, see [74,86,98]

and to assume that the θ_i are logistically distributed with zero mean and variance $\sigma^2 = \pi^2/(3\beta^2)$ over the population. This assumption implies:

$$\lim_{N\to\infty} \frac{1}{N} \sum_i \theta_i = 0 \text{ and } \lim_{N\to\infty} \frac{1}{N} \sum_i H_i = H \tag{6}$$

For a given distribution of choices in the neighbourhood and for a given price, the customer's behaviour is deterministic. An agent buys if :

$$\theta_i > P - H - J_\vartheta \sum_{k\in\vartheta_i} \omega_k, \tag{7}$$

In the full connectivity case (global externality), it is convenient to identify a *marginal customer*, indifferent between buying and not buying. Let $H_m = H + \theta_m$ be his idiosyncratic willingness to pay. This *marginal customer* has zero surplus ($V_m = 0$), that is:

$$\theta_m = P - H - \frac{J}{N-1} \sum_{k\in\vartheta} \omega_k. \tag{8}$$

In this case, an agent buys if: $\theta_i > \theta_m$ and does not buy otherwise.

As underlined by Nadal *et al.* [74] and Phan et al. [86] this model is formally equivalent to a "Random Field Ising Model" (RFIM), intensively studied in statistical physics [41,92], and several variants of it have already been used in the context of socio-economic modeling (Galam et al. [42]; Orléan [78]; Bouchaud [19], Weisbuch and Stauffer [107]). This model, which describes the properties of many different physical systems, has been studied for various network architectures. It is also very interesting for its non equilibrium properties, with avalanches as described below.

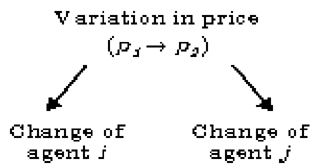
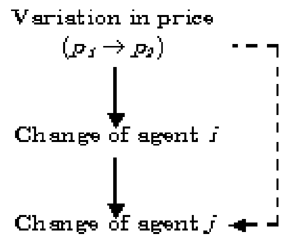
2.4 Local interdependence, avalanches and long range correlation

The term "avalanche" is associated with a chain reaction when the latter is directly induced by modifying the behaviour of one or several other agents and not directly by the variation in price. The price influence is only indirect.

For example, if a price variation (p_1 to p_2) induced a simultaneous but independent change of two agents i and j (connected one to the other or not), the mechanism is directly related to the price and is independent of the network. If on the other hand the price variation induces the behaviour change of agent i, and therefore, because of the behaviour change of agent i (price does not change), agent j changes his behaviour too, in that case the mechanism is an avalanche (domino effect).

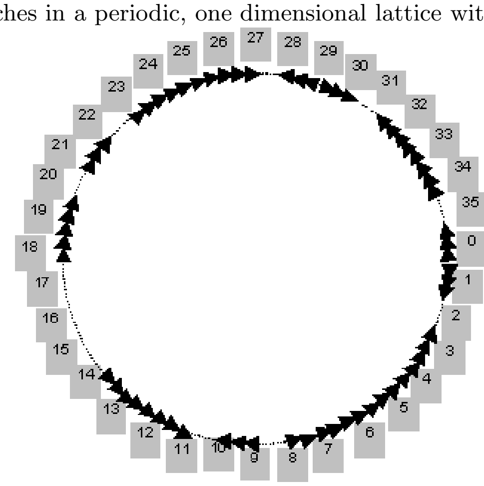
Results of numerical simulations permit us to illustrate the difference between localised avalanches and non-localised avalanches. In a system composed of 36 agents, the evolution of the number of customers is studied for different forms of neighbourhood. In the case where agents are isolated one

Table 4. Direct and indirect effect of prices upon individual choices

Direct effect of price	Indirect effect of price (social influence: avalanche)
	

from the other (Moduleco: "empty" neighbourhood), the dynamic of the system is limited to 36 avalanches made up of only one agent. The social effect is null and the term "avalanche" does not seem very appropriate. If agents are connected to two other agents (Moduleco: "neighbour 2" neighbourhood), the network is a circle. In this numerical simulation, 12 cascades are observed and composed of the following agents: {{15}, {16}, {9, 10}, {14, 13, 12, 11}, {17}, {18, 19, 20}, {8, 7, 6, 5, 4, 3, 2}, {0, 1, 35}, {29, 28, 30}, {23, 24, 25, 26, 27}, {21, 22}, {34, 33, 32, 31}} (see Fig. 2). If the neighbourhood is composed of 4 agents (Moduleco: "neighbour 4"), the numerical simulation shows 10 cascades: {{15}, {16}, {9, 10, 12, 11, 14, 13}, {7, 8}, {5, 6, 4, 3, 2, 0}, {35, 1}, {17, 19, 18, 20}, {29}, {23, 21, 24, 22, 25, 26, 27, 28, 30}, {33, 31, 32, 34}}. In these two cases, the localised effects of the avalanches are very clear because in each one, agents who modify their behaviour are in direct relation with the agent that precedes them.

Fig. 2. Avalanches in a periodic, one dimensional lattice with two neighbours



In the other cases, that is, in the situation where all agents are connected one to the other (Moduleco: "world" neighbourhood), the agent composition of the 8 avalanches is dispersed on the network: {{15}, {9}, {10, 12, 16}, {7}, {0}, {5, 23, 29, 35, 19}, {20, 2, 3, 4, 24, 13, 14, 25, 28, 6, 17, 18, 26, 33, 8, 11, 30, 32, 1, 27, 21}, {22, 34, 31}}. The local interdependence is replaced by a global interdependence.

It should be noted that the size of the largest avalanche is more significant in the last case (21 agents) where all the agents are connected one to the other. In fact, the number of cascades decreases with the size of the neighbourhood, while the size of the largest cascade increases. By widening the interdependence, the communities of agents belonging to the same avalanche tend to join and thus to extend.

The widening of the neighbourhood has several effects when the agent has an average willingness to pay:

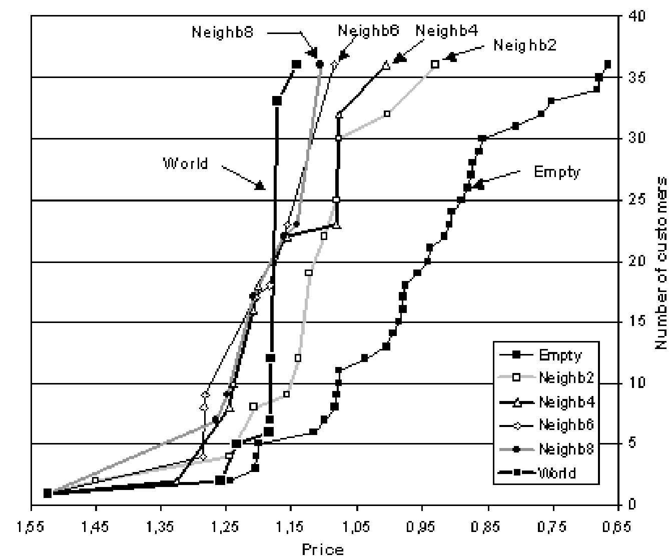
... If, in his initial neighbourhood, he is surrounded by agents with a small willingness to pay, he is likely to purchase the goods late (relatively small price). Increasing the number of neighbours decreases the risk of appearance of this kind of "frozen zone" . The mechanism is similar to that of the threshold of exposure of Valente (1995), mentioned above. In his initial neighbourhood, the agent buys only tardily because he is not exposed enough to the social effect produced by his neighbours.

... If, in his initial neighbourhood, he is surrounded by agents who have a very strong willingness to pay, he will buy the product rapidly (relatively high price). Increasing the size of his neighbourhood decreases the degree of social influence to purchase. Indeed, it is possible that his new neighbours may not have bought the product and come to dilute the very strong influence of the preceding neighbourhood (e.g. if the neighbourhood is composed of two agents who both bought, then the social influence is 100% and falls to 50% if the two new neighbours did not buy it).

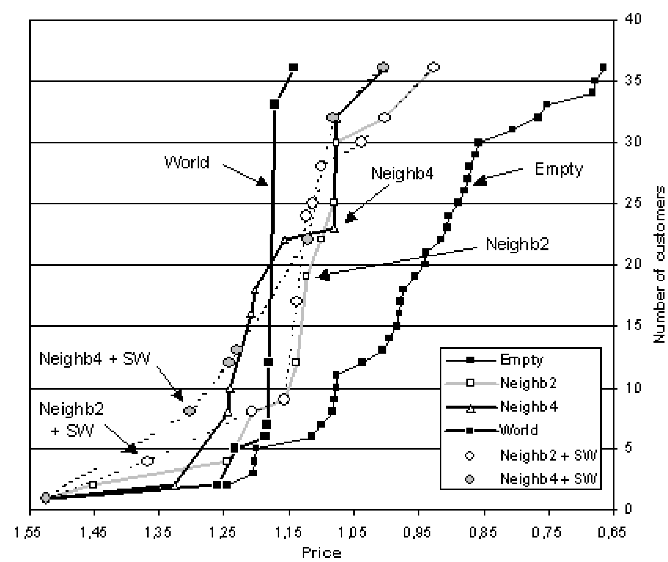
Thus, the negative effects localised on the structure (frozen zone) are less frequent when the size of the neighbourhood increases in a regular network. On the other hand, the local positive effect can be diluted by this widening of the neighbourhood. The distribution of individual characteristics (willingness to pay) and the structural properties of the network of relations will influence the relative importance of these two effects. Thus, to determine the global impact, it is necessary to study the profit evolution according to these characteristics. To isolate the structural effects, this paper concentrates on the effect of the various forms of network (regular network and small-world) with a logistic distribution of individual characteristics.

Figure 3 shows the evolution of the number of customers for several configurations of the network. Figure 3-a shows the positive and negative effects of local interdependence on demand. The line "world" shows the number of customers if there are no particular local relations (all agents are connected one to the other). The situation for isolated agents is represented by the line

Fig. 3. Network structure and the evolution of demand



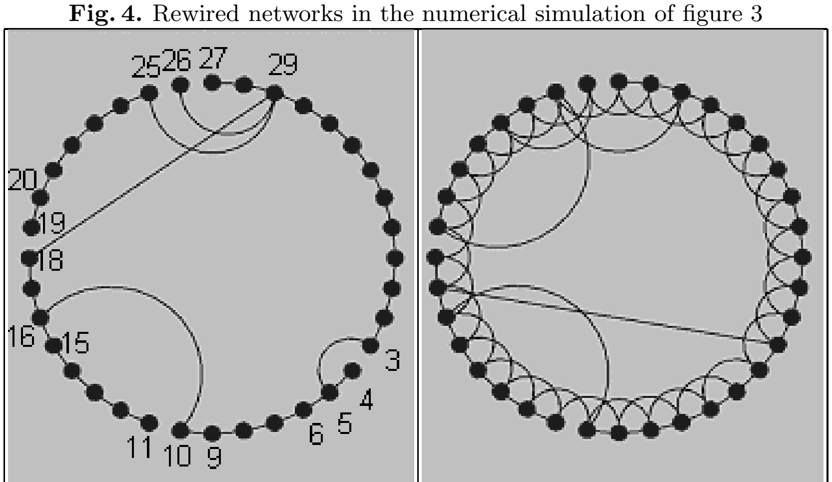
a - regular neighbourhood: positive and negative effect on the demand



b - rewired network (small world): positive and negative effect on the demand "empty". Points to the left of the line " world" correspond to cases where the demand is higher. Conversely, points to the right are associated with lower demand. Therefore, the widening of the neighbourhood has a positive effect on demand for relatively high prices because the number of customers is higher than for the world neighbourhood. The existence of local interdependence has a negative effect for relatively small prices because the number of customers is smaller than for the world neighbourhood. The existence of frozen zones slows down the purchasing process. Nevertheless, the situation remains better in comparison with total independence (empty) and it improves as the neighbourhood is extended.

In figure 3-b, the existence of rewired links between agents (see Figure 4) again improves the number of customers for relatively high prices. This small-world shape has a positive impact on demand because it decreases the negative impact of local frozen zones via long links.

Fig. 4. Rewired networks in the numerical simulation of figure 3



a - Rewired network with neighbour = 2) (5 nodes with 1 link rewired) | b - Rewired network with neighbour = 4

Different effects of the rewired network are detailed in the case of figure 5. The comparison is made with the situation of the regular associated network ("neighbour2"):

The fact that agent 29 concentrates three of the rewired links has a great impact. In the regular network, agent 29 buys at $p = 1.081$, and induces a simultaneous adoption of 28 and 30. Agents 25, 26 and 27 purchase the product at $p = 1.0767$. In the rewired network, the adoption of agent 18 at $p = 1.1389$ (higher price than previously) induces purchasing by agent 29 who induces the simultaneous change of 25, 26 and 28, and then, of agents 24 and 27.

3 Aggregate demand and collective dynamics.

In this class of models, as just seen, the adoption by a single agent in the population (a "direct adopter") may lead to a significant change in the whole population through a chain reaction of "indirect adopters" As a result the aggregate demand dynamics present singular behaviour at the collective level, according to those observed in RFIM studied in statistical physics. This section first reviews such dynamic results obtained by the way of simulations and then provides some analytical features in the special case of "global" externality, which corresponds to the "mean field" approximation in statistical physics.

3.1 Avalanches and hysteresis loops in aggregate demand

In the presence of externality, two different situations - or "phases" - may exist, depending on the price: one with a small fraction of adopters and one with a large fraction. By varying the price, a *transition* may be observed between these phases. The jump in the number of buyers occurs at different price values according to whether the price increases or decreases (*hysteresis*), leading to *hysteresis loops* as presented below.

If the external fields were *uniform*, $H_i = H$, for all i, the model would be equivalent to the classic *Ising* model in an external field: $H - P$. In such a case, one would have a first order transition, with all the population abruptly adopting as H passes through zero from below (and vice versa). In figure 5, the initial (decreasing) price threshold is: $P = H$, where the whole population abruptly adopts. After adoption, the (decreasing) price threshold is: $P = H + J$, where the whole population abruptly leaves the market. When all customers are adopters, price variations between $P = H$ and $P = H + J$ have no effect on demand.

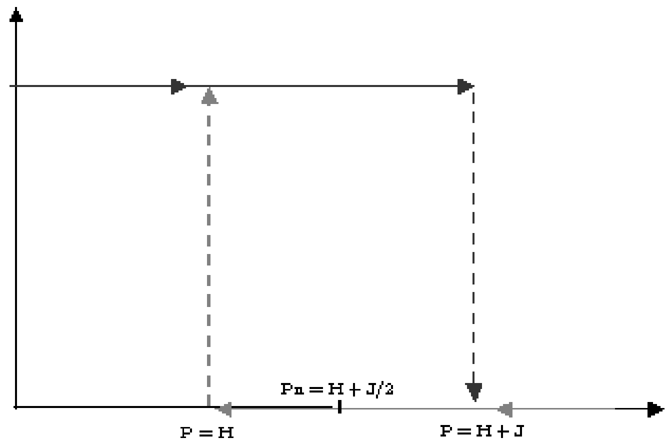
In the presence of *quenched disorder* (non uniform H_i), the number of customers evolves by a series of cluster flips, or avalanches. If the disorder is strong enough (the variance σ^2 of H_i is large - or β is small - compared to the strength of the coupling J), there will be only small avalanches (each agent following his own H_i). If σ^2 is small enough (β large), the phase transition occurs through a unique "infinite" avalanche, like in the uniform case. In intermediate regimes, a distribution of avalanches of all sizes can be observed.

From the theoretical point of view, there is a singular price P_n, which corresponds to the *unbiased* situation, that is, the situation where the willingness to pay is neutral on average: there are as many agents likely to buy as not to buy ($\eta = 1/2$).

Suppose that we start with a network in such a neutral state. Then, on average, the willingness to pay of any agent i is $H_i + J/2 - P$, its average over a set of agents great enough being: $H + J/2 - P$. Thus, the neutral state is obtained for

$$P_n = H + J/2. \tag{9}$$

Fig. 5. Hysteresis with uniform idiosyncratic willingness to pay



For $P < P_n$, there is a net bias in favour of " buy" decisions ($H+J/2-P > 0$), whereas for $P < P_n$, there is a net bias in disfavour of " buy" decisions. The main question for $P = P_n$, is to know whether, in this a priori neutral (unbiased) situation,

. . . either this symmetry will reveal itself in the dynamics: starting from, say, a majority of " buy" decisions, the dynamics will drive the system towards a symmetric state, with as many buyers as non buyers (where essentially every agent follows his own bias, $\omega_i = 1$ if $H_i - p > 0$);

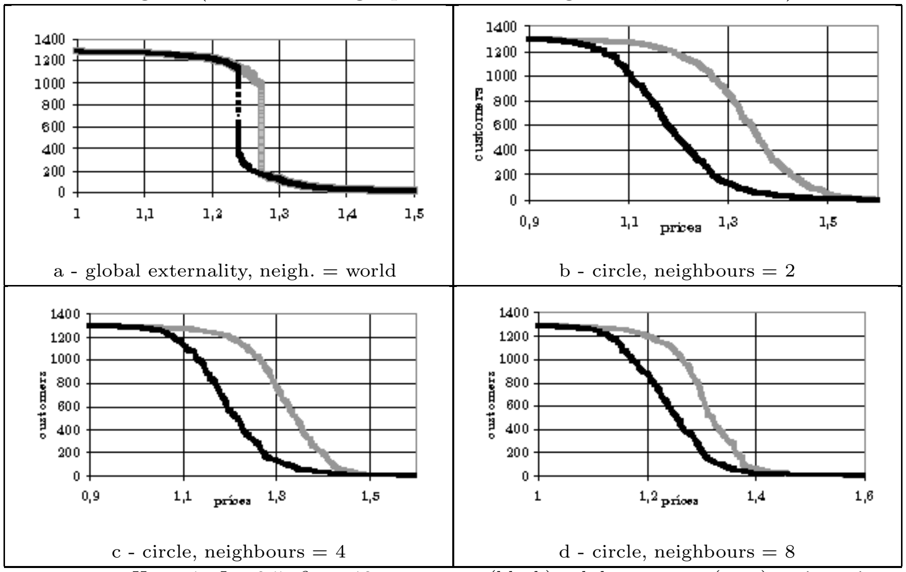
. . . or if there is symmetry breaking, where, e.g. a majority of agent will buy even if one starts with an initial state with as many buyers as non buyers.

One result is that, in the " mean field" analysis (valid for long range interactions - or full connectivity), for a symmetric distribution of the centred idiosyncratic willingness to pay θ_i, one will necessarily observe the first situation if the distribution of the θ_i, has a *maximum* at $\theta_c = 0$, ($H_c = H$), and the second situation may be observed for distributions showing a *minimum* at $\theta_c = 0$. At P not equal to P_n, it is the hysteresis phenomenon which will be the most interesting

It is useful to consider a simple example of a simulation, using the multi-agent framework Moduleco [83–85]. For the simulations presented below, we have $H = 1$ and $J = 0.5$. For a given variation in price, it is possible to observe the resulting variation in demand. The most spectacular result arises in the case of global interactions (complete connectivity) when nearly all agents update their choices simultaneously (synchronous activation regime,Modumeco : "world").

Figure(s) 6 shows the set of equilibrium positions for the whole demand system over all prices, incremented in steps of 10^{-4}, within the interval [0.9, 1.6] under the synchronous activation regime. The relevant parameters are: $H = 1$, $J = 0.5$, $\beta = 10$. One observes a hysteresis phenomenon

Fig. 6. Hysteresis in the trade-off between prices and customers under synchronous activation regime (Moduleco: Logit pseudo-random genrator, seed = 190)



a - global externality, neigh. = world

b - circle, neighbours = 2

c - circle, neighbours = 4

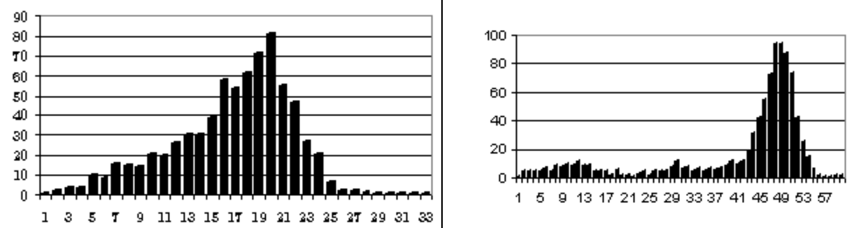
d - circle, neighbours = 8

parameters: $H = 1$, $J = 0.5$, $\beta = 10$ - upstream (black)and downstream (grey) trajectories

with phase transitions around the theoretical point of symmetry breaking: $P_n = H + J/2 = 1.25$. Figure 6a shows the details of straight hysteresis corresponding to the "global" externality (complete connectivity). In this case, the trajectory is no longer gradual, like in the local interdependence case (Figure 6b-d). A succession of growing avalanches arises for $P = 1.2408$, driving the system from an adoption rate of 30% towards an adoption rate of roughly 87%, along the upstream equilibrium trajectory (with decreasing prices). Along the downstream trajectory (with increasing prices) the externality effect induces a strong resistance of the demand system against a decrease in the number of customers. The phase transition threshold is here around $P = 1.2744$. At this threshold, the equilibrium adoption rate decreases dramatically from 73% to 12,7%. Figures 6(b-d) deal with local externality (one-dimensional periodic lattice: the circle case) with 2 to 8 nearest neighbours with the same parameters as in case (a) of global' externality.

Figure 7a shows the chronology of avalanches in the case of the upstream branch of the equilibrium trajectory, for $P = 1.2407$. The evolution follows a smooth path, with a first period of 19 steps, where the initial change of one customer leads to growing avalanches from size 2 to size 81 (6,25% of the whole population). After this maximum, induced changes decrease in 13 steps, including 5 of size one only. Figure 7b shows a different case, with more important avalanches, both in size and in duration (seed 40). The initial impulsion is from a single change for $P = 1.2415$ with a rate of adoption of 19,75%. The first wave includes the first 22 steps, where induced changes increase up to a maximum of 11 and decrease towards a single change. During

Fig. 7. Chronology and sizes of induced adoptions in the avalanche at the phase transition under global externality



a - the avalanche for p = 1.2408 (seed = 190)   b - avalanche for p = 1.2415 (seed = 40)

parameters: $H = 1$, $J = 0.5$, $\beta = 10$ Moduleco: synchronous activation regime).

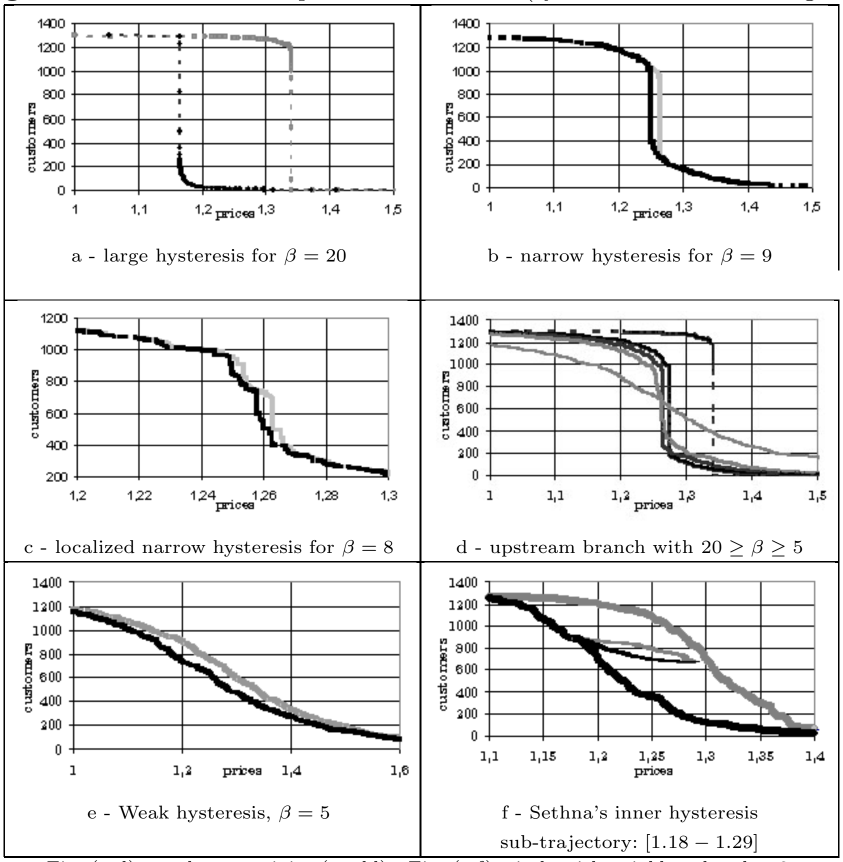
this first sub-period, 124 people change (9,6% of the whole population). After step 22, a new wave arises with a growing size in change towards a maximum of 94 agents both in periods 48 and 49. The total avalanche duration is 60 steps, where 924 induced agent changes arise (71% of the population - 800 in the second wave).

As suggested previously, the steepness of the phase transition increases when the variance $\sigma^2 = \pi^2/(3\,\beta^2)$ of the logistic distribution decreases (that is, increasing β). The closer the preferences of the agents to each other, the greater is the size of avalanches at the phase transition (Figure 8a-c). Figure 8d shows a set of upstream trajectories for different values of β taken between 20 and 5 in the case of global externality. For $\beta = 8$, the scope of the hysteresis is very limited, and finally, for $\beta < 5$ there is no longer any hysteresis at all. Figure 8e shows a narrow hysteresis loop for a regular (periodic) network in dimension one, with eight neighbours, for $\beta = 5$, while Figure 8f exhibits a larger one (see also Figure 6c). Notes that with the same dispersion of agents, one observes weak hysteresis for localised externality, but no hysteresis at all with global externality (in this case with a finite number of agents). Finally, following results by Sethna [92], inner sub-trajectory hysteresis can be observed in the case of this Random Field Ising Model (Figure 8f). Here, starting from a point on the upstream trajectory, an increase in price induces a less than proportional decrease in the number of customers (grey curve). The return to the exact point of departure when the prices decrease back to the initial value (black curve) is an interesting property of Sethna's inner hysteresis phenomenon.

3.2 Demand function for the global externality case (mean field): analytic issues

In this subsection we restrict our investigation to the "global" externality case with homogeneous interactions and full connectivity, which is equivalent

Fig. 8. The trade-off between prices and customers (synchronous activation regime)



a - large hysteresis for $\beta = 20$

b - narrow hysteresis for $\beta = 9$

c - localized narrow hysteresis for $\beta = 8$

d - upstream branch with $20 \geq \beta \geq 5$

e - Weak hysteresis, $\beta = 5$

f - Sethna's inner hysteresis sub-trajectory: $[1.18 - 1.29]$

Fig. (a-d): total connectivity (world) ; Fig. (e-f): circle with neighbourhood = 8

to the mean field theory in physics. Consider the penetration rate η, defined as the fraction of agents that choose to buy at the given price, (i.e. those with $\theta_i > \theta_m$ in 8). In the large N limit, we have $\sum_{k \in \vartheta} \omega_k/(N-1) \approx \eta$, so that: $\theta_m \approx z(\eta)$, where :

$$z(\eta) \equiv P - H - J\,\eta. \tag{10}$$

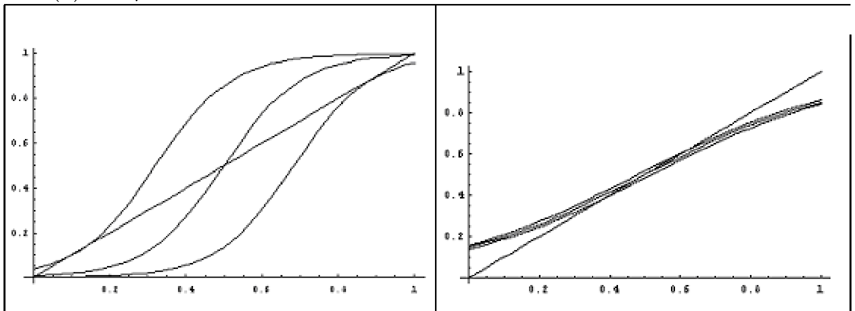
This approximation of (8) allows us to define η as a fixed point:

$$\eta = 1 - F(z(\eta)) \tag{11}$$

where z depends on P, H, and η. Using the logistic distribution for θ_i, we have :

$$\eta = \frac{1}{1 + \exp\left(+\beta\, z\right)} \tag{12}$$

Fig. 9. Fixed points for the penetration rate (market demand for a given price): $1 - F(z)$ vs. η.



a - equation (12) for $\beta = 20$ | b - equation (12) for $\beta = 6$

As just observed with the hysteresis loop in the previous subsection, for given J, H and P, a multiple equilibrium value of η may appear for high values of β (low value of σ). In figure 9a, for $\beta = 20$, we can observe the two equilibria zones, which are included, roughly speaking, between $P = 1.15$ and $P = 1.35$. $P_n = 1.25$ is the unbiased price. Within this zone, we have two equilibria (and resulting hysteresis loop), while for a price lower than 1.15 (a price higher than 1.35), we have a single equilibrium. In figure 9b, $\beta = 6$, we have a single equilibrium for all values of P.

Equation (11) allows us to define the penetration rate (an index of the global demand *in proportion*, without any dimension) as an implicit function of the price

$$\Phi(\eta, P) \equiv \eta + F(P - H - J\,\eta) = 1 \tag{13}$$

$$\eta(P) + F(P - H - J\,\eta(P)) = 1 \tag{14}$$

The shape of this (implicit) demand curve can be evaluated using the implicit derivative theorem:

$$\frac{d\eta(P)}{dP} = \frac{-\partial\Phi/\partial P}{\partial\Phi/\partial\eta} = \frac{-f(z)}{1 - J\,f(z)} \tag{15}$$

where z defined by equation (10), is linked by (14) and $f(z) = dF(z)/dz$ is the probability density.

Given equations (11) and (14) , the global level of demand is :

$$Q^d(P) \equiv N\,\eta(P) \tag{16}$$

The resulting elasticity-price of the demand is not related to size N of the population :

$$-\epsilon(P, \eta) = \frac{d\eta(P)}{dP}\,\frac{P}{\eta(P)} = \frac{-f(z)\,P}{(1 - J\,f(z))\,\eta(P)} \tag{17}$$

Since for a given P, equation (12) finally defines the penetration rate η as a fixed-point, inversion of this equation gives a dimensionless *inverse demand function*:

$$P^d(\eta) = H + J\,\eta + \frac{1}{\beta}\ln\frac{1-\eta}{\eta} \tag{18}$$

4 The distribution of optimal asymptotic prices for a monopolist: first investigations

On the supply side, we consider a monopolist facing heterogeneous customers in a risky situation where this seller has perfect knowledge of the functional form of the agents' surplus functions and their related maximisation behaviour (1). He also knows the size of the population and the statistical (logistic) distribution of the idiosyncratic part of the reservation prices (H_i). But, in the market process, the monopolist cannot observe any of these *individual* reservation prices. He observes only the result of the individual choices (to buy or not to buy). Assume the simplest scenario of "global" externality, where the interactions are the same for all customers, as in equation (4). Thus, hereafter we limit ourselves to this case of full connectivity ($n = N-1$). Then, as just seen with equation (11), the greater N is, the closer to $J\,\eta$ is the social influence on each individual decision. Because the monopolist observes the number of buyer, he also know η (the fraction of customers) in the whole population. As a consequence, in the case of constant marginal cost the monopolist can maximise indifferently the total expected profit or the per unit expected profit, with an expected demand given by equation (11).

4.1 The global externality case

Let C be the monopolist constant cost for each unit sold, so p is his profit *per unit*:

$$p \equiv P - C \tag{19}$$

Since $P - H = (P - C) - (H - C)$, defining:

$$h \equiv H - C, \tag{20}$$

we can rewrite z in (10) and(11) as:

$$z = p \;-\; h \;-\; J\,\eta. \tag{21}$$

Hereafter we write all the equations in terms of p and h.

Since each customer buys a single unit of the good, the monopolist's total expected profit is $p\,N\,\eta$. Thus, in this mean field case, the monopolist's profit is proportional to the total number of customers. He is left with the following maximisation problem:

$$p_M = \arg\max_p \Pi(p), \tag{22}$$

$$\Pi(p) \equiv p\ \eta(p), \tag{23}$$

where $\Pi(p)$ is the per unit expected profit, and $\eta(p)$ is the solution to the implicit equation (11). p_M satisfies: $d\Pi(p)/dp = 0$, which gives:

$$d\eta/dp = -\eta/p \tag{24}$$

Using the implicit derivative (15), we obtain at $p = p_M$:

$$\frac{f(z)}{1 - Jf(z)} = \frac{\eta}{p}, \tag{25}$$

where z, defined in (21), has to be taken at $p = p_M$.

After some manipulations, using equation (17), condition (25) is equivalent to the classical Lener index of monopolist's power:

$$\frac{P - C}{P} = \frac{(1 - Jf(z))\ \eta}{f(z)\ P} = \frac{1}{\epsilon(P, \eta)}, \tag{26}$$

Because the monopolist observes the demand level η, we can use equation (11) to replace $1 - F(z)$ by η. With a logistic distribution, we have : $f(z) = \beta\ F(z)\ (1 - F(z))$; therefore, after some manipulations, equation (25) gives an inverse supply function $p^s(\eta)$:

$$p^s(\eta) = \frac{1}{\beta\ (1 - \eta)} - J\ \eta \tag{27}$$

We obtain p_M and η_M at the intersection between supply (27) and demand (18):

$$p_M = p^s(\eta_M) = p^d(\eta_M). \tag{28}$$

The (possibly local) maxima of the profit are the solutions of (28) for which

$$\frac{d^2\Pi}{dp^2} < 0. \tag{29}$$

After some manipulations, one gets the expression for the second derivative of the profit:

$$\frac{d^2\Pi}{dp^2} = -2\frac{\eta}{p}\left[1 + \frac{2\eta - 1}{2\beta p(1 - \eta)^2}\right] \tag{30}$$

It is clear from this expression that any solution with $\eta > 1/2$ is a local maximum. For $\eta < 1/2$ condition (29) reads

$$\frac{1 - 2\eta}{2\beta p(1 - \eta)^2} < 1. \tag{31}$$

Making use of the above equations, this can also be rewritten as

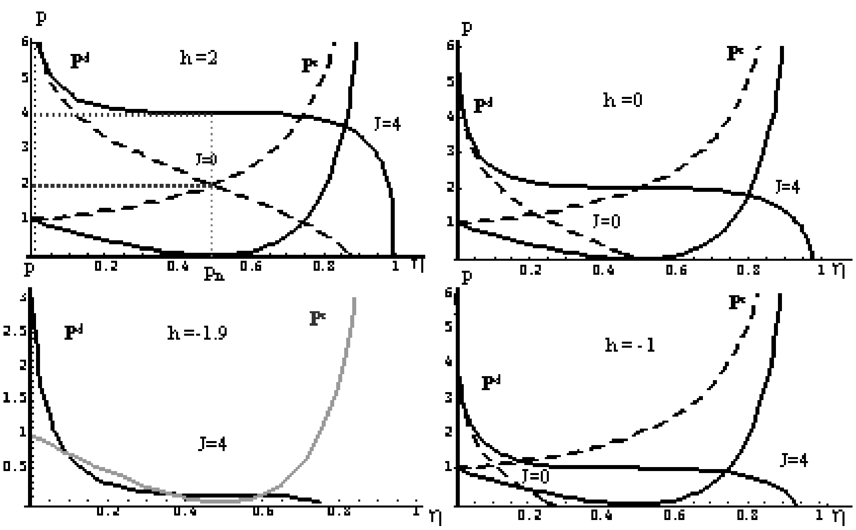
$$2\beta J\eta(1 - \eta)^2 < 1. \tag{32}$$

4.2 Equilibrium analysis and comparative statics

In this section we analyse and discuss the solution of the static supply-demand equilibrium, that is, the solutions of equations (28) and (32).

To illustrate the behaviour of these equations, we represent several examples of inverse supply and demand curves in Figure 10. These curves correspond to different market configurations, obtained for different distributions of the idiosyncratic willingness-to-pay. In the absence of externality ($J = 0$, dashed lines) the case $h = 2$ corresponds to a strong average of the population's willingness-to-pay. The population is neutral for $h = 0$, and $h = -1$ means that, on average, the population is not willing to buy. In all three cases, the supply curve shrinks for increasing values of the external effect J. When the penetration rate is low, the monopolist must lower the price to attract new customers: the second term in the inverse supply function (27) dominates over the first one, and the supply curve bottoms-out. Conversely, when the penetration rate is strong, the positive effects of the externality are dominant and the supply curve grows faster than proportionally to the price decrease. In the same figures we represent the inverse demand and supply curves for the threshold value $\beta J = 4$ (in the figure, $\beta = 1$, $J = 4$), beyond which the demand curve has a minimum at $\eta = 0.5$. For larger values of the external effect, the supply curve is discontinuous, with a decreasing part for low penetration rates and an increasing one for large penetration rates. For $h = -1.9$ (strong aversion to the product) equation (12) has several fixed points, with two stable equilibria.

Fig. 10. Inverse supply and demand curves, for different values of H and J



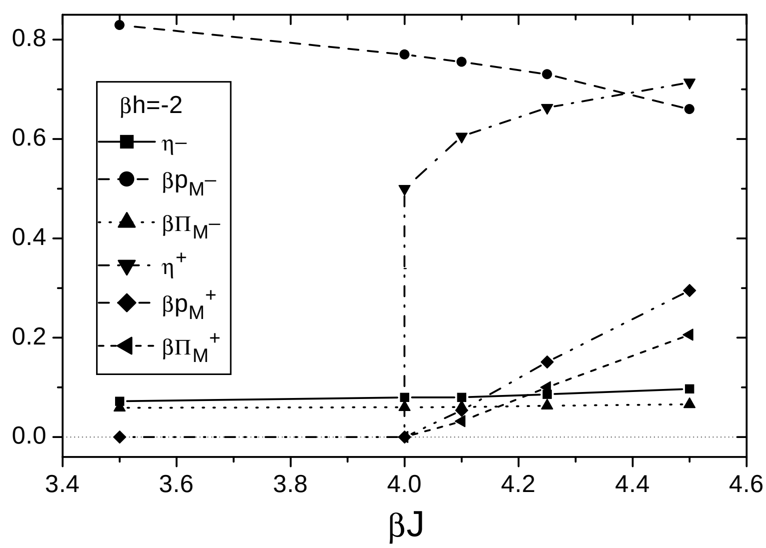
As might be expected, the result for the product βp_M depends only on the two parameters βh and βJ. Indeed, the variance of the idiosyncratic part of the reservation prices fixes the scale of the important parameters, and in particular of the optimal price.

Let us first discuss the case where $h > 0$. It is straightforward to check that in this case there is a single solution η_M. It is interesting to compare the value of p_M with the value p_n corresponding to the neutral situation on the demand side (9). For this, it is convenient to rewrite equation (18) as

$$\beta(p - p_n) = \beta J(\eta - 1/2) + \ln[\eta/(1 - \eta)]. \tag{33}$$

This equation gives $p = p_n$ for $\eta = 0.5$, as it should. For this value of η, equation (27) gives $p = p_n$ only if $\beta(h + J) = 2$: for these values of J and h, the monopolist maximises his profit when the buyers represent half of the population. When $\beta(h + J)$ increases above 2 (decreases below 2), the monopolist's optimal price decreases (increases) and the corresponding fraction of buyers increases (decreases).

Fig. 11. Fraction of buyers η, optimal price βp_M and monopolist profit Π_M, as a function of social influence, for $\beta h = -2$. The upperscripts $-$ and $+$ refer to the two solutions of equations (28) that are relative maxima (Source: [74])



Consider now the case with $h < 0$, that is, on average the population is not willing to buy. Due to the randomness of the individual's reservation price, $H_i = H + \theta_i$, the surplus may be positive but only for a small fraction of the population. Thus, we would expect that the monopolist will maximise his profit by adjusting the price to the preferences of this minority. However, this intuitive conclusion is not supported by the solution to equation (28) when the social influence represented by J is strong enough. The optimal monopolist's strategy shifts abruptly from a regime of high price and a small fraction of buyers to a regime of low price with a large fraction of buyers as βJ increases. This behaviour is analogous to what is called a first order phase transition in physics [93]: the fraction of buyers jumps at a critical value of the control parameter $\beta J_c(\beta h)$ from a low to a high value. Before the transition, above a value $\beta J_-(\beta h) < \beta J_c(\beta h)$ equation (28) already presents several solutions. Two of them are the local maxima of the monopolist's profit function, and one corresponds to a local minimum. The global maximum is the solution corresponding to a high price with few buyers for $\beta J < \beta J_c$, and that of a low price with many buyers for $\beta J > \beta J_c$. Figure 11 presents these results for the particular value $\beta h = -2$, for which it can be shown analytically that $\beta J_- = 4$, and $\beta J_c \approx 4.17$ (determined numerically).

The discussion of the full phase diagram in the plane $\{\beta J, \beta h\}$ is presented in [74].

4.3 Local neighbourhood: the case of $h > 0$ and constrained rewiring

The preceding discussion only considers fully connected systems. The theoretical analysis of systems with finite connectivity is more involved, and requires numerical simulations. The simplest configuration is the one where each customer has only two neighbours, one on each side. The corresponding network is a ring, and has been analysed numerically by way of Moduleco.

Preliminary simulations hold for the case of $h > 0$. Results in Table 5 show that the optimal monopolist's price increases both with the degree of the connectivity graph and the range of the interactions (in particular in the case of small worlds). Different sets of buyers' clusters may form, so that it is no longer possible to describe the externality with a single parameter, like in the mean field case. Further studies in cognitive economics are required in order to explore the possible behaviour of monopolists in such situations.

5 Conclusion

In this paper, we assume a positive (additive) effect of the social influence upon willingness to pay. Heterogeneous agents have a fixed idiosyncratic part in this willingness to pay, unobservable by the monopolist. Numerous models of social interaction often used in economics have an additive random (logistic) part in their willingness to pay [35,21,17,50,86], which corresponds for physicists to a case of 'annealed' disorder. With fixed agent's heterogeneous idiosyncratic characteristic, the model is equivalent to the 'Random field Ising model', belonging to the class of 'quenched' disorder models widely studied by physicists. These two classes of models generally differ, except in the special case of homogeneous interactions with global interactions. In this

Table 5. Average distribution of optimal equilibrium pricing (downstream) over 100 simulations

1296 Agents	optimal price	customers	profit	penetration rate	q
no externality	0,8087	1135	917,91	87,61%	
Neighbour2	1,0259	1239	1 271,17	95,62%	
Neighbour 4	1,0602	1254	1 329,06	96,74%	
N.4 - 130x2	1,0725	1250	1 340,10	96,43%	5%
N.4 - 260x2	1,0810	1244	1 344,66	95,98%	10%
N.4 - 520x2	1,0935	1243	1 358,86	95,89%	20%
N.4 - 780x2	1,0959	1242	1 361,43	95,86%	30%
N.4 - 1296x2	1,1017	1237	1 362,35	95,42%	50%
Neighbour 6	1,0836	1257	1 361,48	96,96%	
N.6 - 130x2	1,0941	1253	1 370,91	96,70%	3%
N.6 - 260x2	1,0997	1252	1 376,78	96,61%	7%
N.6 - 520x2	1,1144	1247	1 389,05	96,19%	13%
N.6 - 780x2	1,1210	1240	1 389,53	95,65%	20%
N.6 - 1296x2	1,1308	1241	1 403,03	95,74%	33%
N.6 - 1296x4	1,1319	1240	1 403,02	95,65%	66%
Neighbour 8	1,1009	1255	1 381,89	96,86%	
Neighbour 8 130 x 2	1,1049	1251	1 381,92	96,52%	3%
N.8 - 260 x 2	1,1169	1249	1 395,43	96,41%	5%
N.8 - 520 x 2	1,1306	1245	1 407,20	96,05%	10%
N.8 - 780 x 2	1,1370	1243	1 413,27	95,92%	15%
N.8 - 1296x2	1,1461	1238	1 419,28	95,56%	25%
N.8 - 1296x4	1,1474	1239	1 421,97	95,63%	50%
N.8 - 1296x6	1,1498	1238	1 423,84	95,56%	75%
world	1,1952	1224	1 462,79	94,44%	

Scale-free Small-world added in an updated version of this paper available at:
http://www-eco.enst-bretagne.fr/~phan/papers/ppn2003.pdf

special situation, which corresponds to *mean-field approximation* in physics, the static (long run) optimal solution is the same in both models [74,86].

In the Random field case, studied here, since the distribution of agents over the network is random, the resulting organisation is complex. ACE Computational Laboratories Moduleco provides a useful and friendly framework for to model, investigate and understand the dynamics of such complex adaptive systems. The strategy followed here is to use ACE as a *complement* to the mathematical theorising, rather than a complete substitute [8].

In the model presented here, the optimal asymptotic monopolist price is known analytically in two polar cases: without externality or with global externality. Analytical results may be possible for the homogeneous regular case, but in more general cases (including the so-called " small world", characterised by both highly local and regular connections and some long range, disordered connections), numerical (statistical) results are often the only possible way. In this preliminary paper, *in silico* experimentation is closer to classical Monte-Carlo simulation than real *cognitive* multi-agent modelling, but this limitation will be overcome in the future. However, present results allow us to observe numerous complex dynamics on the demand side, such as hysteresis, avalanches or Sethna's inner loop hysteresis. As a result, in the case of regular networks and *a fortiori* in the case of small worlds, the seller's problem is generally non trivial, even in the case of risk, where the seller knows all the parameters of the customer's program and the initial distribution of the idiosyncratic parameters.

Given this results, ACE will allow the basic model to be extended in different way beyond this preliminary study, in particular by including belief revision in the agents' capacities, on both the demand and supply side. Interesting challenges in this learning program include network structure learning by the monopolist and evolving networks, dynamic pricing with exploration-exploitation arbitration, which raises the question of the non-stationary environment of both the upstream and downstream trajectory, and Coase conjecture in the case of durable goods (non-repeated buying) among other interesting economic questions.

References

1. Anderson P.W., Stein L., (1983) "Broken Symmetry, Emergent Properties, Dissipative Structure, Life: Are they Related ?", in Anderson P.W (ed.), Basic Notions of Condensed Matter Physics; pp. 263-285.
2. Anderson S.P., De Palma A., Thisse J.-F., (1992) Discrete Choice Theory of Product Differentiation, MIT Press, Cambridge MA.
3. Aoki M., (1996) New Approaches to Macroeconomic Modeling: Evolutionary Stochastic Dynamics, Multiple Equilibria and Externalities as Field Effect; Cambridge University Press.
4. Arthur W.B., (1991) "Designing Economic Agents that Act like Human Agents: A Behavioral Approach to Bounded Rationality", American Economic Review, 81(2), Papers and Prceedings, pp. 353-359.
5. Arthur W. B., (1994) Increasing Returns and Path Dependance in the Economy , University of Michigan Press
6. Arthur B., Durlauf S., Lane D. (eds.) (1997) The Economy as an Evolving Complex System II, Santa Fe Institute Studies in the Sciences of Complexity, Volume XXVII, Addison-Wesley.
7. Auerbach P., (1990) "Market Structure and Firms' Behavior: An Empty Box" , in Arestis P., Kitromilides Y. (eds.), Theory and Policy in Political Economy, Edward Elgar London.
8. Axtell R. (2000a) Why Agents? On carried motivations for agent computing in social sciences WP17 Center on Social and Economic Dynamics The Brooking Institution.
9. Axtell R., (2000) "Effect of Interaction Topology and Activation Regime in Several Multi-Agent Systems", Santa Fe Institute, Working Papers, 00-07-039.
10. Barabsi A.-L., Albert R. (1999) Emergence of scaling in random networks , Science,286, p. 509-512 (1999).
11. Barabsi A.-L., Albert R., Jeong H/ (1999) Mean-field theory for scale-free random networks Physica, 272, p. 173-187.

12. Barthelemy M., Amaral L., Scala A., Stanley H.E., (2000) "Classes of Small-World Networks", PNAS (USA), Cond-mat/0001458.
13. Berge C., (1976) Graphs and Hypergraphs, North-Holland, Amsterdam, 2nd ed.
14. Bikhchandani S., Hirshleifer D., Welch I., (1992) "Theory of Fads, Fashion, Custom and Cultural Change as Informational Cascades", Journal of Political Economy, vol.100(5), pp. 992-1026.
15. Blume L.E., (1993) "The Statistical Mechanics of Strategic Interaction" , Games and Economic Behavior, 5, pp. 387-424.
16. Blume L.E., (1995) "The Statistical Mechanics of Best-Response Strategy Revisions", Games and Economic Behavior, 11, pp. 111-145.
17. Blume L.E., Durlauf S.N., (2001) "The Interaction-Based Approach to Socioeconomic Behavior", in Durlauf S.N., Young P. (eds.), Social dynamics, MIT Press, Cambridge Ma., pp. 15-44.
18. Bornholdt S., Schuster H.G., (eds.) (2003) Handbook of Graphs and Networks, from the Genome to the Internet, Whiley-VCH Weinheim.
19. Bouchaud J.P. (2000) Power-laws in Economics and Finance: some ideas from physics, Quantitative finance 1, 105.
20. Bourgine P., Nadal J.P. (eds.) (2003) Towards a Cognitive Economy, Springer Verlag, Forthcoming.
21. Brock W.A., Durlauf S.N.(2001) Interaction Based Models. In: Heckman, Leamer (eds.) Handbook of Econometrics Volume 5, Ch 54, pp. 3297-3380, Elsevier Science B.V, Amsterdam.
22. Bulow J.I., Geanalopolos J.D., Klemperer P.D. (1985) Multimarket oligopoly: strategic substitutes and complements. Journal of Political Economy, 93/3 p 488-511.
23. Burt R. (1992) Structural Holes : The Social Structure of Competition; Harvard University Press, Cambridge Ma.
24. Cohendet P., Llerena P, Stahn H., Umbauer G. (eds.) (1998) The Economics of Networks, Interactions and Behaviours Springer, Springer Verlag, Berlin.
25. Cont R. (1999) Modeling economic randomness: statistical physics and market phenomena. in Statistical Physics on the Eve of the 21st century: the James B. McGuire Festschrift, p. 47 - 64, Singapore: World Scientific. http://www.cmap.polytechnique.fr//∼rama/papers/festschrift.ps
26. Cont R., Bouchaud J-Ph. (2000) Herd behavior and aggregate fluctuations in financial markets, Macroeconomic dynamics, Vol 4, 170 - 196.
27. Cooper R., John A. (1988) Coordinating coordination failures in keynesian models. Quarterly Journal of Economics, 103 p. 441-463.
28. Cowan R. Jonard N., (2001) "Knowledge Creation, Knowledge Diffusion and Network Structure" in Kirman A., Zimmermann J.B. (eds.), Economies with Heterogeneous Interacting Agents, Lecture Notes in Economics and Mathematical Systems, Vol. 503, Springer, pp. 327-343.
29. Cowan R., Jonard N., Zimmermann J.B., (2001) "The Joint Dynamics of Networks and Knowledge", in Cowan R., Jonard N. (eds.), Heterogenous Agents, Interactions and Economic Performance, Springer, Lecture Notes in Economics and Mathematical Systems, Vol. 521, pp. 155-174.
30. Derrida B., (1986) "Phase Transition in Random Networks of Automata" , in Souletie, Vannimenus, Stora (eds.) Chance and Matter, North-Holland.
31. Dosi G., Marengo L., Fagiolo G., (1996) "Learning in Evolutionary Environments", Santa Fe institute, Working Paper.

32. Durlauf S.N. (1991) "Multiple equilibria and persistence in Aggregate Fluctuations", American Economic Review, 81, p. 70-74.
33. Durlauf S.N. (1993) "Nonergodic economic growth", Review of Economic Studies, vol. 60 no 203, p. 349-366.
34. Durlauf S.N. (1994) "Path Dependance in Aggegate Output", Industrial and Corporate Change, 1, p.149-172.
35. Durlauf S.N., (1997) "Statistical Mechanics Approaches to Socioeconomic Behavior", in Arthur, Durlauf, Lane (eds)., The Economy as an Evolving Complex System II, Santa Fe Institute Studies in the Sciences of Complexity, Volume XXVII, Addison-Wesley, pp. 81-104.
36. Durlauf S.N., (1999) "How can Statistical Mechanics Contribute to Social Science ?", Proceedings of the National Academy of Sciences.
37. Durlauf S.N., Young P. (2001) eds. Social dynamics, The MIT Press, Cambridge Ma. Durlauf S.N., Young P., (2001) (eds)., Social dynamics, MIT Press, Cambridge Ma.
38. Ellison G., (1993) "Learning, Local Interaction, and Coordination" , Econometrica, 61, pp. 1047-71.
39. Favereau O., Lazega E. (eds.) (2003) Conventions and Structures in Economic Organization: Markets, Networks and Organizations, Edward Elgar.
40. Föllmer H., (1974) "Random Economies with many Interacting Agents" , Journal of Mathematical Economics, 1, pp. 51-62
41. Galam S., (1982) "A New Multicritical Point in Anisotropic Magnets. III. Ferromagnets in both a Aandom and a Uniform Longitudinal Field", Journal of Physics C, 15, pp. 529-545.
42. Galam S., Gefen Y., Shapir Y., (1982) "Sociophysics: A Mean Behavior Model for the Process of Strike", Mathematical Journal of Sociology, 9, pp. 1-13.
43. Granovetter M., (1973) "The Strength of Weak Ties", American Journal of Sociology, 78, pp. 1360-1380.
44. Granovetter M., (1978) "Threshold Models of Collective Behavior" , American Journal of Sociology, 83(6), pp. 1360-1380.
45. Granovetter M., (1985) "Economic Action and Social Structure: The Problem of Embeddedness", American Journal of Sociology, 91(3), pp. 1420-1443.
46. Granovetter M., (1992) "Problems of Explanation in Economic Sociology" , in Nohria N. and Eccles R.G. (eds.), Networks and Organizations. Structure, Form, and Action, Harvard Business School Press, Boston, pp. 25-56.
47. Hayek F.A., (1945) "The Use of Knowledge in Society", American Economic Review, pp. 35-4.
48. Hodgson G.M., (1988) Economics and Institutions, Basil Blackwell, Oxford.
49. Holme P., (2001) "Characteristics of Small World Networks" http://www.tp.umu.se/∼holme/seminars/swn.pdf
50. Ioannides Y.M. (1997) "Evolution of Trading Structures", in Arthur, Durlauf, Lane (1997) eds., op. cit. p.129-161.
51. Jonard N., (2002) "On the Survival of Cooperation under Different Matching Schemes", International Game Theory Review, 4, pp. 1-15.
52. Jonard N., Schenk E., Ziegelmeyer A., (2000) "The Dynamics of Imitation in Structured Populations", mimeo, BETA.
53. Kindermann R., Snell J.L., (1980a) Markov Random Fields and their Applications, American Mathematical Society, Providence, Rhode Island.

54. Kindermann R., Snell J.L., (1980b) "On the Relation between Markov Random Fields and Social Networks", Journal of Mathematical Sociology, 7, pp. 1-13.
55. Kirman A.P., (1983) "Communications in Markets: A suggested Approach" , Economic letters, 12, pp. 1-5.
56. Kirman A..P., (1997a) "The Economy as an Interactive System", in Arthur B., Durlauf S., Lane D. (eds.) The Economy as an Evolving Complex System II, Santa Fe Institute Studies in the Sciences of Complexity, Volume XXVII, Addison-Wesley, pp. 491-531.
57. Kirman A.P., (1997b) "The Economy as an Evolving Network", Journal of Evolutionary Economics, 7, pp. 339-353.
58. Kirman A.P., (2001) "Market Organization and Individual Behavior: Evidence from Fish Markets", in Rauch J.E., Casella A. (eds.) Networks and Markets, Russell Sage Pub, pp. 155-195.
59. Kirman A..P., (2003) "Economic Networks", in Bornholdt S., Schuster H.G. (eds.), pp. 273-294.
60. Kirman A.P., Vignes A., (1991) "Price Dispersion: Theoretical Considerations and Empirical Evidence from the Marseilles Fish Market", in Arrow, Kenneth (eds.), Issues in Contemporary Economics, Macmillan, London.
61. Kirman A.P., Vriend N.J., (2001) "Evolving Market Structure: An ACE Model of Price Dispersion and Loyalty", Journal of Economic Dynamics and Control, 25, pp. 459-502.
62. Kirman, A.P. and Zimmermann, J.B. (eds.) (2001) Economies with Heterogeneous Interacting Agents, Lecture Notes in Economics and Mathematical Systems, vol. 503, Springer, pp. 327-343.
63. Lane D., (1993) "Artificial Worlds and Economics part I", Journal of Evolutionary Economics, 3, pp. 89-107.
64. Lane D., (1993) "Artificial Worlds and Economics part II", Journal of Evolutionary Economics, 3, pp. 177-197.
65. Lazega E. (1998)
66. LeBaron B., (2000) "Agent Based Computational Finance: Suggested Readings and Early Research", Journal of Economic Dynamics and Control, 24, pp. 679-702.
67. LeBaron B., (2001) "A Builder's Guide to Agent Based Financial Markets" , Quantitative Finance, 1-2, pp. 254-261.
68. Lesourne J., Orléan A. (eds.) (1998) Advances in Self-Organization and Evolutionay Economics, Economica, Londres.
69. McFadden D. (1974) Econometric Analysis of Qualitative reponse Models. In Griliches Intrilligator (eds.), Handbook of Econometrics Vol II Elsevier Science B.V, Amsterdam.
70. Mantegna R.N., Stanley H.E., (2000) An introduction to Econophysics, Correlations and complexity in Finance, Cambridge University Press.
71. Menard C., (1995) "Market as Institutions versus Organization as Markets ? Disentangling some Fundamental Concepts", Journal of Economic Behavior and Organization, 28, pp. 161-182.
72. Milgram S., (1967) "The Small-World Problem", Psychology Today, 1, pp. 62-67.
73. Mirowski P. Somefun K., (1998) "Markets as Evolving Computational Entities", Journal of Evolutionary Economics, 8, pp. 329-356

74. Nadal J.P., Phan D., Gordon M. B. Vannimenus J.(2003) Monopoly Market with Externality: An Analysis with Statistical Physics and ACE. 8th Annual Workshop on Economics with Heterogeneous Interacting Agents, Kiel,May 29-31, 2003. available at:
http://www-eco.enst-bretagne.fr/∼phan/papers/npgweiha2003.pdf
75. Nadal J.P., Weisbuch G., Chenevez O., Kirman A., (1998) "A Formal Approach to Market Organisation: Choice Functions, Mean Field Approximation and Maximum Entropy Principle . . . ", in Lesourne, Orléan (eds.), pp. 149-159.
76. Nattermann T. (1997) "Theory of the Random Field Ising Model" , cond-mat/9705295.
77. Newman M.E.J. (2000) "Models of Small-World a Review" , cond-mat/0001118v2.
78. Orléan A., (1995) "Bayesian Interactions and Collective Dynamics of Opinion: Herd Behaviour and Mimetic Contagion", Journal of Economic Behavior and Organization, 28, pp. 257-74.
79. Orléan A., (1998a) "Informational Influences and the Ambivalence of Imitation", in Lesourne J. et Orléan A. (eds.), Advances in Self-Organization and Evolutionary Economics, pp. 39-56.
80. Orléan A., (1998b) "The Ambivalent Role of Imitation in Decentralised Collective Learning", in Lazaric N. et Lorenz E. (eds.), Trust and Economic Learning, Elgar Publishers, pp. 124-140.
81. Orléan A. (1998c) "The Evolution of Imitation", in Cohendet P., Llerena P., Stahn H. and Umbhauer G. (eds.) The Economics of Networks. Interaction and Behaviours, Springer-Verlag, p. 325-339.
82. Pajot S., Galam S., (2002) "Coexistence of Opposite Global Social Feelings: The Case of Percolation Driven Insecurity", to appear in International Journal of Modern Physics C, 13.
83. Phan D., (2003) "From Agent-Based Computational Economics towards Cognitive Economics", in Bourgine P., Nadal J.P. (eds.) Towards a Cognitive Economy, Springer Verlag, Forthcoming.
84. Phan D., Beugnard A., (2001a) "Moduleco, a Modular Multi-Agent Plateform, Designed for to Simulate Markets and Organizations, Social Phenomenons and Population Dynamics", Ecole CNRS d'Economie Cognitive, Porquerolles, 25 Sept.-5 oct. 2001.
http://www-eco.enst-bretagne.fr/∼phan/moduleco/english/moduleco00.htm
85. Phan D., Beugnard A., (2001b) "Moduleco, a Multi-Agent Modular Framework for the Simulation of Network Effects and Population Dynamics in Social Sciences, Markets and Organisations", Approches Connexionnistes en Sciences Economiques et de Gestion, 8ème rencontres internationales Rennes IGR, 22-23 novembre.
86. Phan D., Nadal J.P., Gordon M.B. (2003) "Social interactions in economic theory: an insight from statistical mechanics", in Bourgine, Nadal (eds.), Towards a Cognitive Economy, Springer Verlag, Forthcoming.
87. Rauch J.E., Casella A. (eds.) (2001) Networks and Markets, Russell Sage Pub.
88. Schelling T.C., (1960) The Strategy of Conflict, Harvard University Press.
89. Schelling T.C., (1978) Micromotives and Macrobehavior, New York, Norton and Compagny.
90. Schuster H.G., (2001) Complex Adaptive Systems, an Introduction, Scator Verlag, Saarbrücken.

91. Sichman J., Conte R., and Gilbert N. (eds.) (1998) Multi-Agent Systems and Agent-Based Simulation, Springer-Verlag, Berlin.
92. Sethna J.P., Dahmen K., Kartha S., Krumhansl J.A., Roberts B.W., Shore J.D., (1993) "Hysteresis and Hierarchies: Dynamics of Disorder-Driven First-Order Phase Transformations", Physical Review Letters, 70, pp. 3347-3350.
93. Stanley H.E. (1971) *Introduction to phase transitions and critical phenomena*, Oxford University Press.
94. Tesfatsion L., (1997) "How Economists Can Get Alife", in Arthur B., Durlauf S., Lane D. (eds.), The Economy as an Evolving Complex System II, Santa Fe Institute Studies in the Sciences of Complexity, Volume XXVII, Addison-Wesley, pp. 533-564.
95. Tesfatsion L., (2001) "Agent-Based Computational Economics: A Brief Guide to the Literature", in Michie J. (ed.), Reader's Guide to the Social Sciences, Volume 1, Fitzroy-Dearborn, London.
96. Topol R., (1991) "Bubbles and Volatility of Stock Prices: Effect of Mimetic Contagion", Economic Journal, 101, pp. 786-800.
97. Thurstone L.L. (1927) Psychological Analysis. American Journal of Psychology, 38, pp.368-398.
98. Vannimenus J., Gordon M.B., Nadal J.P., Phan D. (2003) "Economic Models of Discrete Choice with Externalities: Properties Derived from Random Field Ising Model", work in progress (draft ENS-LPS).
99. Valente T., (1995) Networks Models of the Diffusion of Innovations, Cresskill, Hampton Press Inc.
100. Vriend N.J., (1995) "Self-Organization of Markets: An Example of a Computational Approach", Computational Economics, 8, pp. 205-231.
101. Watts D.J. (1999) Small Worlds, the Dynamics of Networks between Order and Randomness, Princeton Studies in Complexity, Princeton University Press.
102. Watts D.J. (2003) Six Degrees: The Science of a Connected Age, W.W. Norton & Company
103. Watts D.J. and Strogatz S.H., (1998) "Collective Dynamics of Small-World Networks", Nature, 393, pp. 440-442.
104. Weidlich W., Haag G., (1983) Concepts and Models of a Quantitative Sociology, the Dynamics of Interacting Populations, Springer Verlag, Berlin.
105. Weisbuch G., (1991) Complex Systems Dynamics, Santa Fee Institute Studies in the Sciences of Complexity.
106. Weisbuch G., Kirman A. Herreiner D., (1998) "Market Organisation", in Lesourne J., Orléan A. (eds.) Advances in Self-Organization and Evolutionay Economics, Economica, Londres, pp. 160-182.
107. Weisbuch G, Stauffer D., (2002) "Adjustment and social choice" , cond-mat/0210610.
108. Wilhite A., (2001) "Bilateral Trade and 'Small-World' Networks" , Computational Economics, 18, pp. 49-64.
109. Young P., (1998) Individual Strategy and Social Structure, Princeton University Press.
110. ACE Web site by L. Tesfatsion: http://www.econ.iastate.edu/tesfatsi/ace.htm
111. Econophysics Forum, http://www.unifr.ch/econophysics/
112. Moduleco: http://www-eco.enst-bretagne.fr/∼phan/moduleco/