Complex Behaviours in Binary choice Model with Global or Local social influence

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Abstract. This paper illustrates the effects of global or local social influences upon binary choice. Analytical results are summarized and an ACE (Agent based Computational Economics) approach is used to investigate the corresponding mechanisms of interdependence in the case of a coordination problem and finite size effects.

1 Introduction

In this paper, we explore the effects of the introduction of social influences through fixed interaction structures upon local and global properties of a simple model of binary choice. More specifically, interlinked agents have to make a binary choice. Their preferences are both *intrinsically heterogeneous* (idiosyncratic preferences) and *interactively heterogeneous* (it positively depends on the choice of their neighbours). Aggregate outcomes of such situation may be characterized by multiple equilibria and complex dynamics with "tipping" and "avalanches". The first part of the present study summarizes analytical results in case of global influence while the second part relies on numerical simulations in the case of finite size population for both a global and a local influence network, making use of "Moduleco-Madkit', a multi-agent platform (Gutknecht and Ferber 2000; Phan 2004; Michel *et al.* 2005).

1.1 A short birds eyes view of the literature

The question of binary choices with externalities in the social sciences has been directly addressed by (Schelling 1973, 1978), and the question of individual and collective threshold of adoption has been introduced later by (Granovetter, 1978). In such models, the *individual threshold of adoption* is defined as the number of adopters each agent considers to be sufficient to modify his behaviour. As a result, the final equilibrium depends on the distribution of individual thresholds, and in numerous cases with several equilibria, the selection of a particular equilibrium depends on the history of the collective dynamics. In the context of "global influence", there is no "local network" in the sense that individuals are only sensitive to the percentage of the total population which has previously adopted (a behaviour, a good, a service etc.). (Valente 1995) stresses the importance of the local structure of interpersonal relations in the propagation phenomenon (innovations, opinions), and defines the *threshold of exposure* of an agent as the proportion of adopters in his personal network (neighbourhood) sufficient enough to induce a change in his behaviour.

In the mathematical sociology field, (Weidlich and Haag 1983) proposes, in the global perspective, a generic model of opinion formation based upon a master equation and the Fokker-Plank approximation approach. In the micro-to macro perspective,(Kindermann and Snell 1980) identifies a social network as an application of a Markov random field. (Galam *et al.* 1982) proposes probably the first micro-based application of statistical physics tools to sociology [1]. This pioneering paper proposes a new approach of tipping in collective behaviour applied to strikes. But the scope of this paper is quite larger. Galam and co-authors identify by the way of a "phase analysis" the existence of two regimes (or "phases") separated by a critical point, in the neighbourhood of which the system is extremely sensitive to small changes in parameters as well as to the history of the system. Then, by a tipping effect, small microscopic changes can lead to drastic changes at the macro level.

In economics, the pioneering work of (Föllmer 1974) considers local stochastic interactions by the way of Markov random fields in a general equilibrium model with random preferences. The same year, Gary Becker advocates the introduction of social environment and social interactions in the rational decision of individuals, through his concept of "social income" (Becker 1974). In the middle of the 80's, (Kirman 83) and (Kirman and Oddou and Weber 1986) suggests the use of stochastic graph theory in order to take into account the local communications between agents within the markets. But the real take off for the models of individual choice with interactions and social influence in economics began by the 90's. Some typical contributions are (Brock and Durlauf 2001a), (Glaeser and Sacerdote and Scheinkman, 1996; Glaeser and Scheinkman 2002) for the emphasis on social dimension in a Beckerian tradition, and (Ioannides 2006) for the topologies of interactions [2].

The model briefly discussed in this paper- hereafter referred as the GNP model - was previously presented elsewhere in (Gordon *et al.* 2005), (Nadal *et al.* 2005), (Phan and Semeshenko 2006), and generalized to a large class of distributions in (Gordon *et al.* 2006). The general structure of the GNP model seems to be reminiscent of a class of models by Durlauf and co-authors (Blume and Brock 2001) and especially (Brock and Durlauf 2001a, 2001b) - hereafter referred as the DBB model. But this apparent similarity is only superficial and the structure of GNP and DBB differs by the nature of the disorder (e.g. heterogeneity across agents and randomness). Therefore, in the GNP model agents are heterogeneous with respects to their idiosyncratic preferences, which remain fixed and do not contain additively stochastic term, while the DBB model belongs to the class of both *Random Utility Model*

[1] This approach is qualified as "sociophysics". For a discussion of the relationship with mechanical physics, see (Durlauf 1999), and (Phan and Nadal and Gordon 2004).
[2] see syntheses by (Blume 1997; Durlauf 1997; Blume and Durlauf 2001b)

(RUM) [3] and *quantal choice analysis* (McFadden 1974). The DBB model assumes a double exponential (extreme value, type I) independent identically distributed random variables in each sub-utility of the underlying Thurstone's *discriminal process*, hence the distribution function for the difference of these random variables is logistic. As underlined elsewhere (Phan *et al.* 2004; Nadal *et al.* 2005), these two classes of models are quite different. The DBB model belongs to the class of the Classic Ising Model with "annealed" disorder. The heterogeneity comes from the random term of the RUM only, not from the deterministic term, assumed to be the same for all agents On the contrary, our own model is formally equivalent to a "Random Field Ising Model" (RFIM), with a fixed heterogeneous idiosyncratic term: the disorder is said to be "quenched" (i.e. there is no random utility). These two kinds of models can lead to very different behaviours (Stanley 1971; Galam and Aharony 1980, 1981; Galam 1982; Sethna *et al.* 1993, 2005). Some of them are presented below.

2 The Model and its global behaviour

The question of social influence over individual choice is now on the economist's agenda. In this section, some analytical results from the GNP model are presented and discussed in the particular case of global influence and symmetric triangular distribution of idiosyncratic preferences (Phan and Semeshenko 2006).

2.1 Modelling the individual choice in a social context

We consider a set of N agents $i \in \Lambda_N \equiv \{1, 2, .., N\}$ with a classical linear willingness-to-adopt function. Each agent makes a simple binary choice, that is, either adopts $(\omega_i = 1)$ or does not adopt $(\omega_i = 0)$. A rational agent chooses ω_i in the strategic set $\Omega \equiv \{0, 1\}$ in order to maximize a linear surplus function $\omega_i V_i$:

$$\begin{gathered} W_i\left(\omega_i \left|\tilde{\omega}_{-i}\right.\right) \equiv \max_{\omega_i \in \{0,1\}} \left\{\omega_i . V_i\left(\tilde{\omega}_{-i}\right)\right\} \\ \text{with}: \quad V_i\left(\tilde{\omega}_{-i}\right) = \left(H_i - C\right) + \tfrac{J_{ik}}{N_{\vartheta i}} \sum_{k \in \vartheta_i} \tilde{\omega}_k \end{gathered} \tag{1}$$

Where C is the cost of adoption [4] and H_i represents the idiosyncratic preference component. Some other agents k, influence agent i's preferences through their own choices ω_k. Agents k hereafter called neighbours of i are within a subset: $\vartheta_i \in \Lambda_N$, of size $N_{\vartheta i}$, called neighbourhood of i such that each agent $k \in \vartheta_i$. This social influence is represented here by a weighted sum of these choices. Let us denote $J_{ik}/N_{\vartheta i}$ the corresponding weight i.e. the marginal social influence on agent i, from the decision of agent $k \in \vartheta_i$. This social influence is assumed to be positive: $J_{ik} > 0$. For simplicity, we consider here only the case of homogeneous influences, that is,

[3] Originated in Thurstone's model of comparative judgment (Thurstone 1927), introduced in economics by (Marschak 1960; Block and Marschak 1960), see also (Mansky 1977)

[4] i.e. the price to buy one unit in the market case or some common cost in the non market case, cf. (Granovetter 1978, Glaeser and Scheinkman 2002)

identical positive weights for all influence parameters in the neighbourhood: $\forall i \in \Lambda_N, \forall k \in \vartheta_i : J_{ik} = J$. For a given neighbour k taken in the neighbourhood ϑ_i, the marginal social influence is $J/N_{\vartheta i}$ if the neighbour is an adopter ($\omega_i = 1$), and zero otherwise. The individual surplus (1) can be rewritten in a more simply way as:

$$\begin{aligned} W_i\left(\omega_i \left|\tilde{\omega}_{-i}\right.\right) &\equiv \max_{\omega_i \in \{0,1\}} \left\{\omega_i \left(H_i - C + J\eta_i^e(\tilde{\omega}_{-i})\right)\right\} \\ \text{with :} \quad & \eta_i^e(\tilde{\omega}_{-i}) \equiv \sum_{k \in \vartheta_i} \tilde{\omega}_k / N_{\vartheta i} \end{aligned} \tag{2}$$

Where $\eta_i^e(\tilde{\omega}_{-i})$ is the expected rate of adoption within the neighbourhood of i. In the GNP model, the private idiosyncratic term H_i, is assumed invariable in time, but may differ from one agent to the other. It is useful to introduce the following notation for H_i - hereafter called *Idiosyncratic Willingness to Adopt* (IWA):

$$H_i = H + Y_i \ \text{ with : } \lim_{N\to\infty} \frac{1}{N} \sum_N Y_i = 0 \ \Rightarrow \ \lim_{N\to\infty} \frac{1}{N} \sum_N H_i = H \tag{3}$$

where $Y i$ is the outcome of an i.i.d. random variable Y with zero mean, distributed among the agents. Let $f_y(Y)$ be the Probability Density Function (pdf) of Y. As Y_i remains fixed, the resulting distribution of agents over the network of relations is a random field. Then, this model is formally equivalent to a "*Random Field Ising Model*" (RFIM) and the disorder is said to be "quenched" (i.e. there is no stochastic term). Therefore, agent's choices are purely deterministic (in contrast with the random utility approach in the DBB model, as mentioned before). An example of such model in sociophysics literature is (Galam 1997).

It is possible to relate our own model of binary choice with social influence to game theoretic models. Under our assumptions, all the agents have the same form of instrumental rationality (then, best response with respect to theirs expectations $\tilde{\omega}_{-i}$) and each agent has only two possible strategies: $\omega_i \in \Omega$. It is possible to represent the *total payoff* of an agent by the "normal form" matrix $G1$. From this standpoint, player 2 is a fictitious player; say a kind of *Neighbourhood Representative Player* (NRP), who stands for the behaviour of the neighbourhood as a whole. If every k in the neighbourhood plays $\omega_k = 0$, the NRP plays the pure strategy $\omega_{nr} = 0$.

Table 1. Payoff matrix for an agent i and best reply equivalent potential game

(a) - game G1	$\omega_{nr} = 0$	$\omega_{nr} = 1$	(b) - game G2	$\omega_{nr} = 0$	$\omega_{nr} = 1$
$\omega_i = 0$	0	0	$\omega_i = 0$	$C - H_i$	0
$\omega_i = 1$	$H_i - C$	$H_i - C + J$	$\omega_i = 1$	0	$H_i - C + J$

Player i in rows - fictitious NRP - indexed nr - in columns

Conversely, if every k in the neighbourhood plays $\omega_k = 1$, the NRP plays the pure strategy $\omega_{nr} = 1$. However, the classical framework of two players game theory does not apply in numerous cases, because the strategic set of the player i and the NRP is generally asymmetric. Player i must plays only a pure strategy, while NRP can plays a mixed strategy. That is, the expected rate of adoption within the neighbourhood $\eta_i^e(\tilde{\omega}_{-i})$ corresponds to the expected share of ($\omega_k = 1$) players in the neighbourhood. Consequently, player i plays his best response against the mixed strategy $\eta_i^e(\tilde{\omega}_{-i})$. Then, this later interpretation of the mixed strategy can be related to the framework of population games (Blume, 1997), where agent i plays in $N_{\vartheta i}$ bilateral confrontations against all agents k in their neighbourhood, with the payoff matrix G1' based on average payoff ($(H_i - C)/N_{\vartheta i}$ for $\omega_i = 1$ against $\omega_k = 0$ and $(H_i - C + J)/N_{\vartheta i}$ for ($\omega_i = 1$) against ($\omega_k = 1$), respectively, zero otherwise). Indeed, since $N_{\vartheta i}$ is fixed, maximising the total surplus or the average surplus lead to the same solution. One may add a constant term to one column and multiply all the columns by a constant term (here N_{ϑ_i}) without affecting the dominance ordering analysis, hence the best reply outcome. Thus, the following matrix in Table 1.b is said to be "best reply equivalent" to the one of Table 1.a This means notably that the Nash equilibria are the same whether one considers Game G1 (Table 1.a) or Game G1' (Table 1.b). This class of game with "best reply equivalence" (hence, similar Nash equilibrium) is called a (weighted) potential games (Monderer, Shapley, 1996). For agents (type 0) such as: $H_i - C + J < 0$, strategy $\omega_i = 0$ (never adopt) is stricly dominant. Conversely, for agents (type 1) such as: $H_i - C > 0$, strategy $\omega_i = 1$ (always adopt) is strictly dominant. Then, the relevant situation is one with agents (type 3) such as $H_i - C < 0$ and $H_i - C + J > 0$. In this case, the choice depends on $\eta_i^e(\tilde{\omega}_{-i})$, the expected rate of adoption within the neighbourhood. If all agents are of type (3), we have typically a coordination game with two Nash equilibrium; the so called "Stag Hunt Game". With bounded support for Y, $[Y_{min}, Y_{max}]$, this is the case if: $Y_{max} \leq C - H \leq Y_{min} + J$, what implies a sufficiently strength intensity of social effect, with respect to the dispersion of preferences $J \geq Y_{max} - Y_{min}$.

2.2 Individual interactions and chain reaction

Table 2. A typology of interactions and demand dynamics

Neighbourhood	(a) No relations	(b) Localised relations	(c) Generalised relations
Level of interactions	(independent agents)	Localized interactions	Global interactions
sensitivity to the network topology	Null	Strong	Null
Avalanches	No	localised in the network	not localised in the network

In the first extreme case (a), there are no relations between agents. In this case, the aggregate demand depends on any interaction structure, and there is no external effect (local or global). The agents are independent one from each other. In the second extreme case (c), all agents interact by means of global interactions (e.g. the rate of adoption in the whole population). Let $\eta \equiv N_a/N$ be te rate of adoption within the population. For N sufficiently large, this rate is closed to the rate of adoption within the neighbourhood of each agents (full connectivity) say: $\eta \simeq N_a/(N-1)$. This case corresponds to the *means field approximation* in statistical physics. All agents are equivalent in the network. In this way, the aggregate demand is sensitive to the global external effect, but remains independent of the topology of the network (because the neighbourhood of each agent is composed of all the other agents). Thus, finite sequences of interdependent decisions called "avalanches" may arise, but such “dominoes effects” are *not localised* in the network of interactions and depend only on individual IWA, given the *global* rate of adoption, whatever the local rate of adoption (i.e. localized in the near neighbourhood). Finally, the intermediate case (b) corresponds to situations where agents have specified relations reified by the way of some network topology (regular neighbourhood or not). Interactions between agents are local, and the topology of the interpersonal network matters. This local interdependence may give rise to *localised avalanches* on the network (Table 2).

The term *avalanche* is associated with a chain reaction where the latter is directly induced by the behavioural modification of one or several other agents and not directly by the variation in cost. The cost influence is only indirect. For example in the left part of the (Table 3), an external cost variation (the same for all agents: C to C') induces a simultaneous (but independent of all social influence) change of two agents i and j (connected one to the other or not). Thus, the mechanism is directly related to the cost and independent of the social network. If, on the other hand, the cost variation induces the behavioural change of agent i, and therefore, because of agent i changes his behaviour, then agent j changes also his behaviour by social effect without any new change in cost, by “domino effect”. In that case, the cumulative effect of a chain of such induced influences is called an “avalanche”.

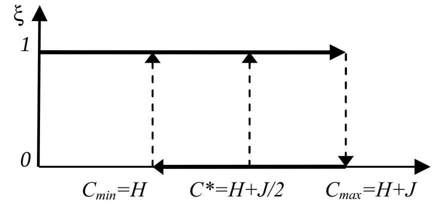
Table 3. Direct and indirect effect of prices upon individual choices

Direct effect of price	Indirect effect of price (social influence: avalanche)
Variation in cost $(C \longrightarrow C')$ ↙ ↘ Change of agent i / Change of agent j	Variation in cost $(C \longrightarrow C')$ ↓ Change of agent i ↓ Change of agent j

2.3 Avalanches and hysteresis loops in aggregate behaviour with unique IWA

In this class of models, the adoption by a single “direct adopter” may lead to a significant change in the whole population through a chain reaction of “indirect adopters”. The jump in the number of adopters occurs at different cost values according to whether the costs increases or decreases, leading to *hysteresis loops* as presented below. If the IWA is the same for all agents, ($H_i = H$, for all i), the model would be equivalent to the (quenched) Classic Ising Model with an “uniform external field”: $H - C$. In such a case, one would have a so called “first order transition”, with all the population abruptly adopting as $H \geq C$. In Figure 1, this initial (decreasing) threshold is: $C_{min} = H$, where the whole population abruptly adopts. After adoption, the (increasing) cost threshold is: $C_{max} = H + J$, where the whole population abruptly choose $\omega_i = 0$ (for all i). When all agents are adopters, cost variations between: C_{min} and C_{max} have no effect on the agents choice. Within that zone $[C_{min}, C_{max}]$, there are two possible equilibria for a given cost.

Fig. 1. Hysteresis with unique IWA ($H_i = H$)



From a theoretical point of view, there is a singular cost $C^* = H + J/2$ (the center of the interval $[C_{min}, C_{max}]$), which corresponds to the *unbiased situation*, where the willingness to adopt is neutral on average. Suppose that we start within such a neutral state. The agents makes their initial choice on the basis of some prior expectation about the number of adopters and further choice by updating this prior by use of the observed outcome. Assume first that all agents have the same expectation $\eta_i^e = \eta^e$ for all i. Then, each agent has a willingness to adopt equal to: $H + J\eta^e - C^* = J(\eta^e - 0.5)$. If $\eta^e > 1/2$, the expected surplus is positive and all agents adopt. Then, the ex post surplus will be $J/2$. Conversely, if: $\eta^e < 1/2$, the expected surplus is negative and no agent adopts. The final result is similar if we have two classes of people with heterogeneous expectations. Those with $\eta_i^{e+} > 1/2$ (in proportion α) adopt. If $\alpha > 1/2$, the percentage of adopters is such as pessimistic agents with $\eta_i^{e-} < 1/2$ but $\eta_i^{e-}\alpha > 1/2$ also adopt, and so on until complete adoption (and inverse process for $\alpha < 1/2$). This critical point plays a central role in the so called *spontaneous symmetry breaking*, even when agents are only locally connected. As in our simple example, the collective equilibrium state becomes identical to the individual state: either all agents adopt, or no agent adopts (Galam, 2004)

3 Avalanches and hysteresis with global and local interactions in finite-size population

This model describes the properties of many different systems (physical as well as social). It has been studied for various network architectures. In the presence of externality, and depending on the parameters, two different stable equilibria - or "phases" - may exist for a given cost: one with a small fraction of adopters (in some cases with no adopter) and one with a large fraction (in some cases, everybody adopts). By an external variation of the cost, a transition may be observed between these phases. Next subsection concerns the case of infinite size population and global interaction, while last subsection deals with both local and global interactions, by the way of computer simulations and finite size population.

3.1 Equilibrium analysis: phase diagram with global externality

In order to present equilibrium results, let us consider now the special case of global externality from a static standpoint (e.g. without expectations). In this case, the individual surplus function (1) can be rewritten simplier as a function of the equilibrium value of the rate of adoption η.

$$W_i\left(\omega_i\left|\eta\right.\right) \equiv \max_{\omega_i \in \{0,1\}} \left\{\omega_i\left(Y_i + H - C + J\eta\right)\right\} \tag{4}$$

It is convenient to identify the *marginal adopter*, indifferent between adopting and not adopting. Let $H_m = H + Y_m$ be his idiosyncratic willingness to adopt (IWA). This marginal adopter has zero surplus $W_m = V_m = 0,\ \forall \omega_m \in \Omega$, that is:

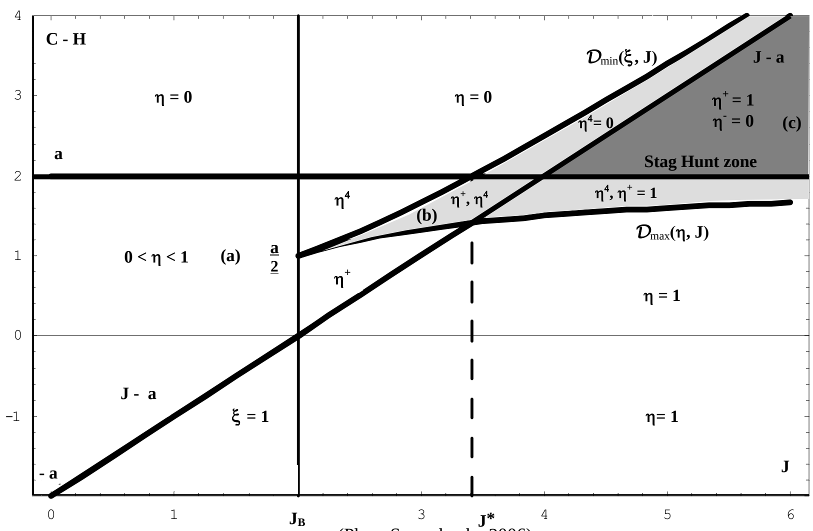
$$Y_m = C - H - J\eta \tag{5}$$

Consequently, an agent adopts if $Y_i > Y_m$ and does not adopt otherwise. Then, if the law of Y has a continuous pdf, the rate of adoption is the solution of the following:

$$\eta = P\left(Y_i > Y_m\right) \equiv G_y(Y_m) \equiv \int_{Y_m}^{\infty} f_y\left(y\right) dy \tag{6}$$

More specifically, assume that Y follows a symmetric triangular law, with bounded support $[-a, +a]$. The fixed point condition (6) has one or three solutions, with two stable equilibria in this later case (Phan, Semeshenko 2006). More generally, this is a generic property of this model of binary choice with externality for a large class of distribution (Gordon *et al.* 2006).

Fig. 2. Equilibrium regimes in the phase spaces: $(J, C - H, a = 2)$

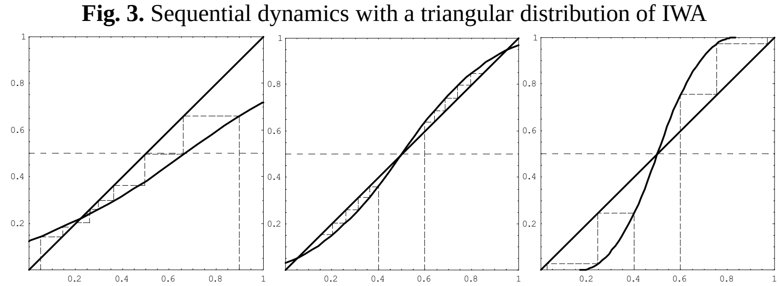


source: (Phan, Semeshenko 2006)

According to a methodology introduced by the Physicists, Figure 2 exhibits in the phase plane $(J, C - H)$ a cartography of regions with one equilibrium or two equilibria, with respect to the value of corresponding parameters. For the detail of the calculus for this triangular case, see (Phan, Semeshenko, 2006). A stable equilibrium can be viewed as a Nash equilibrium of a population game (section 2.1). For low cost and sufficiently strength of social coupling, everybody adopt (in zone south and south east on the phase diagram). Conversely, for high cost and weak social coupling, nobody adopt (North West). In the west and south west, for $J < J_B$ and $J - a < C - H < a$, there is a *polymorphic Nash equilibrium* with both non-adopters and adopters in proportion $0 < \eta < 1$ (Figure 3.a). Let $\mathcal{D}(\eta, j) \equiv G^{-1}(\eta) - J\eta$. For $J > J_B$, there is a zone with two stable Nash equilibria (the grey zone on Figure 2). This zone is delimited by two frontiers given by $\mathcal{D}_{min}(\eta, j) = J - a + a^2/(2J)$ and $\mathcal{D}_{max}(\eta, j) = a - a^2/(2J)$. Therefore, if: $\mathcal{D}_{min}(\eta, j) > C - H > \mathcal{D}_{max}(\eta, j)$ there are two equilibria: one rate of adoption less than 50% (possibly 0%) and other more than 50% (possibly 100%) .

Accordingly, the polymorphic single equilibrium zone has two extensions for $J^* > J > J_B$, with: $0 < \eta^- < 0.5$ if: $\mathcal{D}_{min}(\eta, j) < C - H < a$ and: $0 < \eta^+ < 0.5$ if: $J - a < C - H < \mathcal{D}_{max}(\eta, j)$ respectively. In the darker grey zone, the strength of social coupling is such as $J > 2a$ and therefore: $a \leq C - H \leq J - a$. According to section 2.1 all agents are of type (3), and we have a “Stag Hunt” coordination game with two equilibria, one without any adoption and another with complete adoption.

Fig. 3. Sequential dynamics with a triangular distribution of IWA



(a) - $J = 1.5,\ \ C - H = 1$; $\eta = \frac{1}{4.5} \simeq 0.22$; (b) - $J = 3,\ \ C - H = 1.5$; $\eta^- = 0.056, \eta^+ = 0.944$; (c) - $J = 6,\ \ C - H = 3$ $\eta^- = 0, \eta^+ = 1$

source: (Phan, Semeshenko 2006)

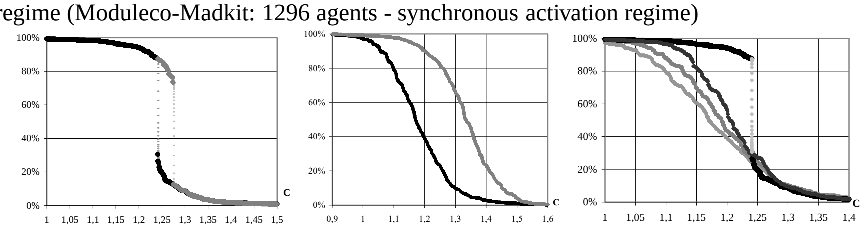
In order to illustrate some typical equilibrium cases from the phase diagram, let us consider a recurrent relation drawn from the fixed point condition (6) in the case of the agents have identical myopic expectations: $\eta^e(t) = \eta(t-1)$. Then, $Y_m(t) = C - H - J\eta(t-1)$. This recurrent relation allows us to represent agents' learning by a graphic of fixed point dynamics on Figure 3. In Figure 3.a the stable equilibrium is unique, while there are two stable equilibria separate by an instable fixed point on Figure 3.b (polymorphic) and Figure 3.c (Stag Hunt).

3.2 Avalanches and hysteresis loops in aggregate behaviour with logistic IWA

The previous results concern the case of infinite size population and global interaction. This section is devoted to the case of a finite size population by the way of computer-based simulations. From work in progress, we present some sample experiments with both global and local interactions on a *random field* with logistic quenched disorder, representing idiosyncratic fixed IWA.

In the presence of a quenched disorder, the number of customers may evolve by a serie of cluster flips, or avalanches. If the disorder is strong enough (i.e. the variance σ^2 of Y is large with respect to the strength of the social coupling J), there will be only small avalanches (There are numerous agents following their own H_i). If σ^2 is small enough, the phase transition occurs through a unique "infinite" avalanche, similar to the case with the unique H for all agents (section 2.3). This is called a "first order phase transition" by physicists. In intermediate regimes, a distribution of smaller avalanches of various sizes can be observed. It is useful to consider as exemple a sample of a simulation, using the multi-agent framework Moduleco-Madkit (Gutknecht and Ferber 2000; Phan 2003; Michel *et al.* 2005) [5].

Fig. 4. Hysteresis in the trade-off between cost and adopters under synchronous activation regime (Moduleco-Madkit: 1296 agents - synchronous activation regime)



a - Global externality (TC) b - Circle, Neighbours = 2 c - upstream branch: circle and TC

$H = 1, J = 0.5, \beta = 10 \Rightarrow \sigma = \pi.\sqrt{3}/10 \simeq 0.544$, Logit pseudo-random generator, seed = 190

(a-b) upstream (black)and downstream (grey) trajectories (c) circle with $N = 2, 4, 8$ and TC: from grey to black

Figure(s) 4.a-c shows for a set of particular experience with the same distribution of IWA (seed = 190). Points are equilibrium rate of adoption for the whole system for cost incremented in steps of 10^{-4} under the synchronous activation regime (all agents update their behaviour at the same time). One observes a hysteresis phenomenon with phase transitions around the theoretical point of symmetry breaking: $C^* = H + J/2 = 1.25$. Figures 4.a deals with the "global" externality, while Figure 4.b corresponds to a "local" externality (on a one-dimensional periodic lattice: the circle case, with two nearest neighbours) with the same parameters and IWA distribution in both cases. Figure 4.d shows the upstream branch (decreasing costs) of a circle with nearest neighbours ($N = 2, 4, 8$) and the same global externality case (TC) than on Figure(s) 4.a. Figure 4.a shows the details of straight hysteresis corresponding to the "global" externality (complete connectivity). In this case, the trajectory is no longer gradual, like in the local interdependence case on Figure 4.b. Along the upstream equilibrium trajectory (with decreasing costs) an avalanche arises for $C = 1.2408$, by a succession of cluster flips, driving the system from an adoption rate of 30% towards an adoption rate of roughly 87%. Along the downstream trajectory (with increasing costs) the externality effect induces a strong resistance of the

[5] For the simulations presented below, we have a logistic distribution where $\beta = \pi.\sqrt{3}/\sigma$ is the logistic parameter, $H = 1, J = 1/2$ and $\beta = 10$; $(J.\beta = 5)$

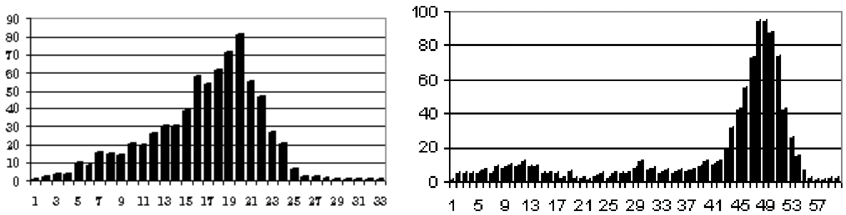
system against a decrease in the number of adopters. The phase transition threshold is here around $C = 1.2744$. At this threshold, the equilibrium adoption rate decreases dramatically from 73 % to 12.7 %.

The *threshold of exposure* (TE) is the proportion of adopters in the local neighbourhood of an agent sufficient enough to induce a change in his behaviour (Valente 1995). For finite neighbourhood, this TE evolves by discrete jump and therefore it is very sensitive to the size of the neighbourhood. This threshold effect may be either favourable or unfavourable to adoption, depending of the relative position of the agent with respect to the *unbiased situation*. For instance let $J = 1$ and $C = 1$. The unbiased situation is such as: $C^* - H = J/2$, hence $H = 0.5$. If $H_i = 0,4$ (the agent i is below the unbiased situation), then $C - H_i = 0.6$; for $N = 2$ the TE_2 is 2, say 100%; while $TE_4 = 3$ (75%) with $N = 4$ and $TE_8 = 5$ (62,5%) with $N = 8$. Thus, in this case, the relative TE (i.e. the rate of the TE over the neighbourhood) decreases with the widening of the neighbourhood. Conversely, if the IWA is such as the agent is above the unbiased situation say, $H_i = 0,6$: then $C - H_i = 0.4$, the TE_2 is 1 (50%) for $N = 2$. This rate remains the same with $N = 4$ ($TE_4 = 2$) and with $N = 8$ ($TE_8 = 4$). In this later case, we need a neighbourhood equal or superior to $N = 10$ in order to reduce the relative TE below the relative threshold of 50%. The finite size effect of the TE is then both discontinuous and asymmetric.

For finite size population and finite neighbourhood, the equilibria distribution is very sensitive to the possibility of local clusters both with higher or lower adoption with respect to the mean field case (complete connectivity or social influence). This is related to both the discrete distribution of the thresholds and the possibility of extreme situation (where an agent is surrounded by neighbours all with either a small or a great IWA). Such effect is more sensitive for low cost / high degree of adoption, where the adoption is slower with local neighbourhood, due to the existence of clusters of non-adopters, called “frozen zone”. Figure 4.c shows the evolution of the rate of adoption for several configurations of the network: one dimensional periodic (circle) with near-neighbourhood of size 2, 4, 8 and complete connectivity (from light grey to black respectively). In this case with 1296 agents, the negative effect of local interdependence is clearer than the positive one (for low rates). In the case under consideration, the widening of the neighbourhood has a little positive effect on adoption. For relatively high cost / low rates of adoption, the number of adopters is higher than for the full connectivity. For relatively small costs (high rates of adoption), the existence of local interdependences (frozen zone) has a strong negative effect, hence the number of customers is clearly smaller than in the case of global influence, but this later effect is little attenuated by the widening of the neighbourhood effect.

In the case of a finite size sample, there is some local irregularities in the discrete distribution of characteristics (IWP), even with “near-perfect” pseudo-random generator. Therefore, the shape of an avalanche is completely dependent on the realizations of Y_i. Then, gaps in the ordered sequence of the Y_i produce fluctuations in the chronology of induced adoptions, as well as possible multi-modal shape, like in Figure 5.b. Despite the non-generic properties of such figures, this kind of historic profile remains relevant for empirical experiments in finite size situations.

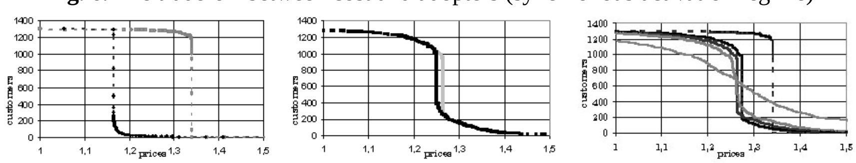
Fig. 5. Examples of chronology and sizes of induced adoptions in the avalanche at the phase transition under global externality in two single experiences with 1296 agents



a - the avalanche for p = 1.2408 (seed = 190) b - avalanche for p = 1.2415 (seed = 40)
parameters: $H = 1$, $J = 0.5$, $\beta = 10$ Moduleco: synchronous activation regime).

Figure 5.a shows the chronology of an avalanche in the case of the upstream branch of the equilibrium trajectory, for $C = 1.2407$. The evolution follows a smooth path, with a first period of 19 steps, where the initial change of one customer leads to growing induced effects from size 2 to size 81 (6.25 % of the whole population). After this maximum, induced changes decrease in 13 steps, including 5 of size one only. Figure 5.b shows a different case, with more important induced effects, both in size and in duration (seed 40). The initial impulsion is from a single change for $C = 1.2415$ with a rate of adoption of 19.75 %. The first wave includes the first 22 steps, where induced changes increase up to a maximum of 11 and decrease towards a single change. During this first sub-period, 124 agents change (9.6 % of the whole population). After step 22, a new wave arises with a growing size in change towards a maximum of 94 agents both in step 48 and 49. The total avalanche duration is 60 steps, where 924 induced agent changes arise (71 % of the population - 800 in the second wave). As suggested previously, the steepness of the phase transition increases when the variance σ^2 of the logistic distribution decreases (increasing β).

Fig. 6. The trade-off between cost and adopters (synchronous activation regime)



a - large hysteresis for $\beta = 20$ b - narrow hysteresis for $\beta = 9$ c - upstream branch with $20 \geq \beta \geq 5$
(a - c): total connectivity (TC)

The closer the preference of the agents to each other, the greater the size of avalanches at the phase transition (Figures 6.a-b). Figure 6.c shows a set of upstream trajectories for different values of β taken between 20 and 5 ($10 \geq J\beta \geq 5$), in the case of global externality. The scope of the hysteresis decreases with β ; for $\beta < 5$, there is no longer any hysteresis at all (remark that intermediate positions in straight hysteresis are transitory equlibibrium (in light grey in 4.a) and finite size effect, and do not appear in the analytical case with “infinite” population).

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