Seller's dilemma due to social interactions between customers

Mirta B. Gordon (1), Jean-Pierre Nadal (2),
Denis Phan(3) and Jean Vannimenus (2)

(1) Laboratoire Leibniz-IMAG,
46, Ave. Félix Viallet, 38031 Grenoble Cedex 1, France
(mirta.gordon@imag.fr,
http://www-leibniz.imag.fr/Apprentissage/Membres/Gordon/index.html)
(2) Laboratoire de Physique Statistique, Ecole Normale Supérieure,
24 rue Lhomond, 75231 Paris cedex 05, France
(nadal@lps.ens.fr, http://www.lps.ens.fr/~nadal)
(3) CREM, Université de Rennes 1, France
(denis.phan@univ-rennes1.fr, http://perso.univ-rennes1.fr/denis.phan/)

Abstract

In this paper we consider a discrete choice model where heterogeneous agents are subject to mutual influences. We explore some consequences on the market's behaviour, in the simplest case of a uniform willingness to pay distribution. We exhibit a first order phase transition in the profit optimisation by the monopolist: if the social influence is strong enough, there is a regime where, if the mean willingness to pay increases, or if the production costs decrease, the optimal solution for the monopolist jumps from a solution with a high price and a small number of buyers, to a solution with a low price and a large number of buyers. Depending on the path of prices adjustments by the monopolist, simulations show hysteretic effects on the fraction of buyers.

1 Introduction

In this paper we explore the effects of social interactions on the properties of a simple market model, in which the individuals have to make a binary choice (whether to buy or not a single unit of a good) given a price fixed by a single seller (a *monopolist*). Each individual has a reservation price, *i.e.* the maximum price he is ready to pay for the good, which is the sum of two terms: an idiosyncratic willingness-to-pay (IWP), and a social component proportional to the fraction of his neighbours that buy. This last term, known in the economics litterature as an *externality*, is the result of mutual interactions between customers. As a consequence, the market may present complex behaviours [1, 2]. As we show in the following, these interactions introduce multiple solutions in the demand function and are responsible of the existence of a transition in the optimal strategy of the monopolist.

There is a straightforward analogy between the customers description and the Ising model, which has been pointed out in recent papers in economics [3, 4, 5, 6, 7]. Depending on the nature of the IWPs, the analogy corresponds to two different families of models in statistical mechanics: either the IWPs are randomly chosen and remain fixed, or they present independent temporal fluctuations around a fixed (homogeneous) value. The former case corresponds to a Random Field Ising Model model (*quenched disorder*). If the distribution of the temporal fluctuations in the latter is logistic, it corresponds to an *annealed*, that is thermal, disorder.

On the supply side, we assume that the monopolist does not know the IWP of each customer, but is aware of its distribution among the population. He determines the price that optimizes his profit. Since the demand may be a multiple valued function of the price, the monopolist's situation is risky.

In this paper we consider the case of quenched disorder in which the distribution of the IWPs is uniform and the social influence is global. The latter assumption is equivalent to the mean field approximation, and allows us to obtain analytic results. We determine the supply and demand curves, and the phase diagram, as a function of the average IWP of the population, and of the social influence strength.

2 Simple models of discrete choice with social influence

We consider a set Ω_N of N agents with a classical linear IWP function [8]. Each agent $i \in \Omega_N$ either buys ($\omega_i = 1$) or not ($\omega_i = 0$) one unit of the single given good in the market. A rational agent chooses ω_i in order to maximize his *surplus function* V_i:

$$\max_{\omega_i \in \{0,1\}} V_i = \max_{\omega_i \in \{0,1\}} \omega_i (H_i + \sum_{k \in \vartheta_i} J_{ik} \omega_k - P), \tag{1}$$

where P is the price of one unit and H_i represents the idiosyncratic preference component. Some other agents k, within a subset $\vartheta_i \subset \Omega_N$, such that $k \in \vartheta_i$, hereafter called neighbours of i, influence agent i's preferences through their own choices ω_k. This social influence is represented here by a weighted sum of these choices. Let us denote J_{ik} the corresponding weight i.e. the marginal social influence on agent i, of the decision of agent $k \in \vartheta_i$. When this social influence is assumed to be positive ($J_{ik} > 0$), it is possible, following Durlauf [4], to identify this external effect as a *strategic complementarity* in agents' choices [9].

For simplicity we consider here only the case of *homogeneous* influences, that is, identical positive weights $J_{ik} = J_\vartheta$ and identical neighbourhood structures ϑ of size n, for all the agents. That is,

$$J_{ik} = J_\vartheta \equiv J/n > 0 \quad \forall i \in \Omega_N, \;\; k \in \vartheta_i, \tag{2}$$

2.1 Psychological *versus* economic points of view

Depending on the nature of the idiosyncratic term H_i, the discrete choice model (1) may represent two quite different situations. Following the typology proposed by Anderson et al. [10], we distinguish a *psychological* and an *economic* approach to individual choices. Within the psychological perspective (Thurstone [11]), the utility has a *stochastic* aspect because "there are some qualitative fluctuations from one occasion to the next... for a given stimulus". In this case, the IWP present independent temporal fluctuations around a fixed (homogeneous) value (this point of view will be referred to hereafter as the *TP-model*). If the distribution of these temporal fluctuations is logistic, it corresponds to an *annealed disorder*, that is, to finite temperature [7, 12]

On the contrary, within the *economic* perspective of McFadden [13] (see Anderson et al. [10]), each agent has a willingness to pay that is *invariable* in time, at least during the period under consideration, but may differ from one agent to the other (we call hereafter this perspective the *McF model*). This situation is known in the Physics litterature as a model with *quenched disorder*. Even if a seller knows the statistical distribution of the IWP over the population, he cannot observe each specific individual IWP. In the langague of interactive decision theory, this seller is in a em risky situation.

Thus, these two perspectives, which differ in the nature of the individual willingness to pay, correspond to very different theoretical models.

In the TP model, the idiosyncratic preference has two sub-components: a constant deterministic term H (the same for all the agents), and a time- and agent-dependent additive term $\epsilon_i(t)$ ($H_i = H + \epsilon_i$). The $\epsilon_i(t)$ are i.i.d. random variables of zero mean; in the simulations they are refreshed at each time step (asynchronous updating). Agent i decides *to buy* according to the conditional probability

$$P(\omega_i = 1 | z_i(P, H)) = \mathcal{P}(\epsilon_i > z_i(P, H)) = 1 - F(z_i(P, H)), \tag{3}$$

with

$$z_i(P, H) = P - H - J_\vartheta \sum_{k \in \vartheta_i} \omega_k, \tag{4}$$

where $F(z_i) = \mathcal{P}(\epsilon_i \leq z_i)$ is the cumulative distribution of the random variables ϵ_i. In the standard TP model, the agents make repeated choices, and the time varying components $\epsilon_i(t)$ are drawn at each time t from a logistic distribution with zero mean, and variance $\sigma^2 = \pi^2/(3\beta^2)$, $F_L(z) = (1 + \exp(-\beta\, z))^{-1}$.

In the McF model, the private idiosyncratic terms H_i are randomly distributed over the agents, but remain fixed during the period under consideration. There are no temporal variations: the ϵ_i are strictly zero. In analogy with the TP model, it is useful to introduce the following notation: $H_i = H + \theta_i$. In the limit of a very large number of customers, this implies:

$$\lim_{N \to \infty} \frac{1}{N} \sum_i \theta_i = 0 \;\text{ and }\; \lim_{N \to \infty} \frac{1}{N} \sum_i H_i = H. \tag{5}$$

The customer's behaviour, given the price and the choices in his neighbourhood ϑ_i, is deterministic. Agent i buys if:

$$\theta_i > P - H - J_\vartheta \sum_{k \in \vartheta_i} \omega_k. \tag{6}$$

If the θ_i are logistically distributed with zero mean and variance $\sigma^2 = \pi^2/(3\beta^2)$ over the population, then the correspondence between the TP and the McF models is better the larger the number of agents, but it is actually strict only in the limit of an infinite population. Notice however that, although formulated originally for a logistic IWP distribution, the McF model may be generalized to any distribution. In the present paper we restrict our investigation to the McF model and we illustrate its behaviour in the particular case where the IWPs are uniformly distributed over the population.

2.2 Static versus dynamic points of view

Hereafter, we concentrate on the McF model in the "global" externality case, considering *homogeneous interactions* and *full connectivity*, which is equivalent to the *mean field* model at zero temperature in physics.

Within this general framework, we are interested in two different aspects. First we consider a static point of view computing the set of possible economic equilibria, solving for the equality between demand and supply. This will allow us to analyse in section 4 the optimal strategy of the monopolist, as a function of the model parameters.

We consider next the market's dynamics assuming the usual parallel MonteCarlo updating rules, which in the models of economic agents correspond to myopic learning with full information (i.e. based on the last iteration, without memory): based on the observation of the behaviour of the other agents at time $t-1$, each agent decides at time t to buy or not to buy. We show that, in general, the market converges towards the static equilibria of the preceding section, except for a precise range of the parameter values where interesting static as well as dynamic features are observed.

In Physics, these two kinds of analysis correspond to the study of, respectively, the thermal equilibrium properties within the *statistical ensemble* framework on one side, and the out of equilibrium dynamics (which, in most cases, approaches the static equilibrium through a relaxation process) on the other side.

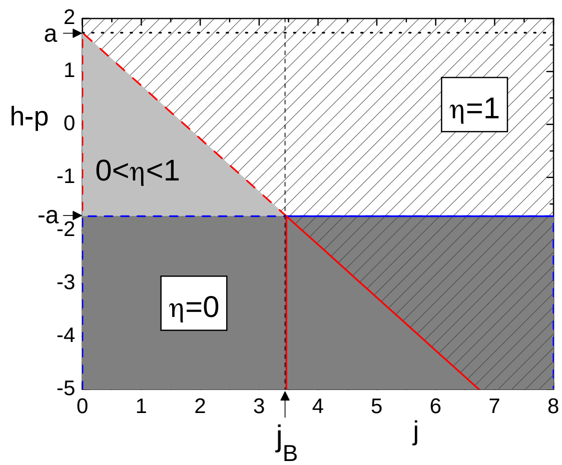

Figure 1: Customer's phase diagram in the plane $(j, h-p)$: The region with diagonal stripes corresponds to parameters for which all the customers are buyers ($\eta = 1$). The dark grey region corresponds to parameter values hindering adoption ($\eta = 0$). For $j < j_B \equiv 2a$, there are parameters for which the fraction of buyers grows linearly with $h-p$ (light grey region). For $j > j_B$ the latter solution does not exist, and there is a range of values of $h-p$ for which the two solutions ($\eta = 1$ and $\eta = 0$) coexist.

3 Aggregate demand

As discussed in the preceding section, we consider the full connectivity case in the limit of a very large number of agents. *The penetration rate* η defined as the fraction of customers that choose to buy at a given price, $\eta \equiv lim_{N\to\infty} \sum_{k=1}^{N} \omega_k/N$, can be approximated by the social influence term of the agents' surplus function (eq. (1)): $\eta \approx \sum_{k\in\vartheta} \omega_k/(N-1)$. In the large N limit, equation (6) may thus be replaced by

$$\theta_i > P - H - J\,\eta, \tag{7}$$

where θ_i is a random variable.

For the following discussion it is convenient to divide both sides of the preceding equation by σ, the variance of the distribution, and consider normalized variables

$$\begin{aligned} x_i &\equiv \theta_i/\sigma, \\ p &\equiv P/\sigma, \\ h &\equiv H/\sigma, \\ j &\equiv J/\sigma. \end{aligned} \tag{8}$$

Then, the distribution of x_i has zero mean and unitary variance. In the illustrations we consider the uniform distribution defined by

$$f(x) = \begin{cases} 1/2a & \text{if} - a \le x \le a \\ 0 & \text{otherwise} \end{cases} \tag{9}$$

with $a = \sqrt{3}$.

Let us identify the *marginal customer*, indifferent between buying and not buying. Let $h_m = h + z$ be his IWP. This *marginal customer* has zero surplus ($V_m = 0$), that is:

$$z \equiv p - h - j\,\eta. \tag{10}$$

Thus, an agent i buys if $x_i > z$, and does not buy otherwise. Equations (7) and (10) allow to obtain η as a fixed point:

$$\eta = 1 - F(z) \tag{11}$$

where z, defined by equation (10), depends on p, h, j and η.

Since for a given p, equation (11) defines the penetration rate η as a fixed-point, inversion of this equation gives an *inverse demand function*:

$$p^d(\eta) = h + j\,\eta + G(\eta) \tag{12}$$

where $G(\eta)$ is the inverse of the complementary distribution function; it satisfies:

$$\int_{G(\eta)}^{\infty} f(x)dx = \eta. \tag{13}$$

Thus, $G(\eta)$ is a non increasing function of η. Notice that its derivative satisfies $G'(\eta) = -1/f(G(\eta))$. Given values of j and h, for most values of p, (11) has a unique solution $\eta(p)$. These are the values of η that satisfy $p = p^d(\eta)$ where $p^d(\eta)$ is given by equation (12).

However, for j larger than a *critical value* j_B, that depends on the specific distribution $f(x)$ there is a range of prices such that, for any value of p within this range, (11) has multiple solutions. More precisely, if $f(x)$ is monomodal (like in the present case), there are two stable solutions and an unstable one, and $j_B = 1/f_{Max}$, where f_{Max} is the maximum value of $f(x)$. The unstable solutions are those with positive derivative of $p^d(\eta)$ (they correspond to states where the fraction of buyers would *increase* upon increasing the price).

The upper and lower values, p_1 and p_2, of the range presenting multiple demand solutions,

$$p_1(j,h) < p < p_2(j,h) \tag{14}$$

are obtained from the condition that eq. (12) has a single degenerate solution $\eta(p)$. For differentiable pdfs, it is:

$$\eta = 1 - F(z), \text{ and } \frac{d(1-F(z))}{d\eta} = 1.$$

The corresponding penetration depths, η_1 and η_2 define the limiting inverse demands prices

$$p_l = h + j\eta_l + G(\eta_l)\ ; \quad l \in \{1,2\}. \tag{15}$$

Note that these limiting prices are not necessarily positive. A negative price gives the measure of how much the costs should be lowered, or the average willingness to pay h increased, for the monopolist be able to sell the good.

In the case of the uniform distribution, which is not differentiable and has a finite support, the above analysis becomes very simple. The critical value of j is $j_B^u \equiv 2a$, where the superscript stands for *uniform*. For $j < 2a$, the inverse demand curve (12) is a non increasing function of η, but for $j > 2a$ it becomes non monotonic. The fact that j_B^u is *independent* of η is a degeneracy specific to the uniform distribution, and does not arise with smoother pdfs.

These results are summarized on figure 1, which presents the customer's phase diagram for the uniform distribution.

It is interesting to note that the set of equilibria is the same as what would be obtained if agents had rational expectations about the choices of the others: if every agent knew the distribution of the x_i, he could compute the equilibrium state compatible with the maximisation of his own surplus -taking into account that every agent does the same-, and take his decision (to buy/not to buy) accordingly. For $j < j_B$ every agent could thus anticipate the value of η to be realized at the price p, and make his choice according to (7). For $j > j_B$, however, if the price is set within the interval $[p_1, p_2]$, the agents are unable to anticipate which equilibrium will be realized, even though the one with the largest value of η should be preferred by every one (since it is a Pareto dominating equilibrium). In game theory, this situation is typical of coordination games [14].

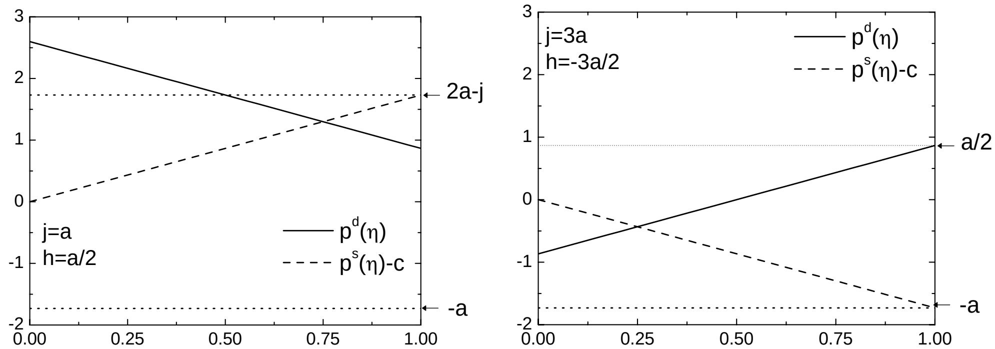

Figure 2: Inverse supply and demand curves for different values of h and j.

4 Supply side

On the supply side, we consider a monopolist facing heterogeneous customers in a risky situation where the monopolist has perfect knowledge of the functional form of the agents' surplus functions and their maximisation behaviour (1). He also knows the statistical distribution of the idiosyncratic part of the reservation prices, $h + x_i$. However, the monopolist cannot observe any *individual* reservation price. He only observes the aggregate result of the individual choices (to buy or not to buy), that is, the fraction of customers η, whose expected value for a given price is given by equation (11). Notice that, as the interactions are global, in the limit of a large number of customers (rigorously for $N \to \infty$) this is the same quantity as the one that enters in the term of social influence among customers, equation (7).

4.1 Profit maximisation

Let $c \equiv C/\sigma$ be the monopolist's cost in units of σ (the variance of the distribution of the IWP) for each unit sold, so that $p - c$ is his (normalized) profit *per unit*.

Since each customer buys a single unit of the good, the monopolist's total expected profit is $(p - c)\ N\ \eta$. He is left with the following maximisation problem:

$$p^M = \arg\max_p \Pi(p), \tag{16}$$

where $N\ \Pi(p)$ is the expected profit, with:

$$\Pi(p) \equiv (p - c)\ \eta(p), \tag{17}$$

and $\eta(p)$ is the solution to the implicit equation (11). If there is no discontinuity in the demand curve $\eta(p)$ (hence for $j \leq j_B$), p^M satisfies $d\Pi(p)/dp = 0$, which gives $d\eta/dp = -\eta/p$ at $p = p^M$. Using the implicit equation (11) to calculate the derivative, we obtain at $p = p^M$:

$$\frac{f(z)}{1 - Jf(z)} = \frac{\eta}{p}, \tag{18}$$

where z has to be taken at $p = p^M$.

Because the monopolist observes the demand level η, we can use equation (11) to replace $1 - F(z)$ by η. After some manipulations, equation (18) gives the monopolist's price as a function of the demand, which may be interpreted as an (effective) inverse supply function $p^s(\eta)$:

$$p^s(\eta) = c - \eta[G'(\eta) + j]. \tag{19}$$

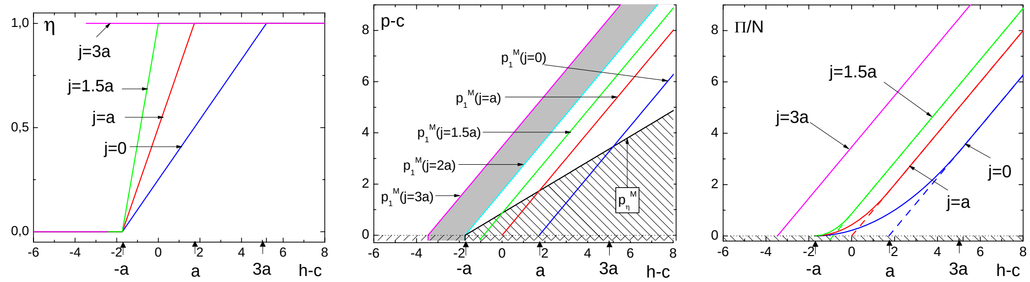

Figure 3: Fraction of buyers η, optimal price p^M and monopolist's profit Π_M, as a function of the average willingness to pay $h-c$ for different values of the social influence weight j. *Left*: For $j = 3a > j_B$ the fraction of buyers has two solutions ($\eta = 0$ and $\eta = 1$ for a range of values of $h-c$. *Center*: The line p^M_η shows the optimal prices for $j < j_B$. In the hashed region, these are larger than the prices (shown under the hashes) allowing full adoption ($\eta = 1$), but they maximize the profit. *Right*: Profits. Dashed lines are the profits corresponding to the full adoption prices.

We obtain p^M and η^M, the corresponding fraction of buyers, as the intersection between supply (19) and demand (12):

$$p^M = p^s(\eta^M) = p^d(\eta^M). \tag{20}$$

The monopolist's supply price is the solution of (20) which maximizes his profit. If $f(x)$ is differentiable, the maximum satisfies

$$\frac{d^2\Pi}{dp^2} < 0. \tag{21}$$

In the case of multiple extrema, clearly the one which maximises Π has to be selected.

For $j > j_B$, the monopolist has to find $p = p^M$ which realises the program:

$$p^M : \max\{\Pi_-(p^M_-), \Pi_+(p^M_+)\} \tag{22}$$

$$p^M_+ = arg \max_p \Pi_+(p) \equiv p\,\eta_+(p), \tag{23}$$

$$p^M_- = arg \max_p \Pi_-(p) \equiv p\,\eta_-(p) \tag{24}$$

where the subscript $+$ $(-)$ refers to the solution of (11) with the largest (smallest) fraction of buyers.

Figures 2 present two cases of inverse demand and supply functions for the uniform distribution. The two solutions for $j > j_B \equiv 2a$ here are the extreme values $\eta_+ = 1$ and $\eta_- = 0$, at the boundaries of the $[0, 1]$ interval, but for smoother pdfs, these are generally inside this interval.

4.2 Phase diagram of the monopolist's strategy

In this section we discuss the optimal supply-demand static equilibria, that is, the solutions of equation (20) that maximize the monopolist's profit, in the case of the uniform distribution. As might be expected, the result for p^M depends only on the two parameters $h-c$ and j. As already seen, the variance of the idiosyncratic part of the reservation prices fixes the scale of the parameters, and in particular of the optimal price. The calculation is straightforward; hereafter we summarize the results, presented on figures 3 and 4.

For $j < j_B$, the fraction of customers is a single valued function of the price. The actual penetration rate is thus determined by the price posted by the monopolist. In this region there are two (relative) maxima of the profit, one of which has a low price and allows full

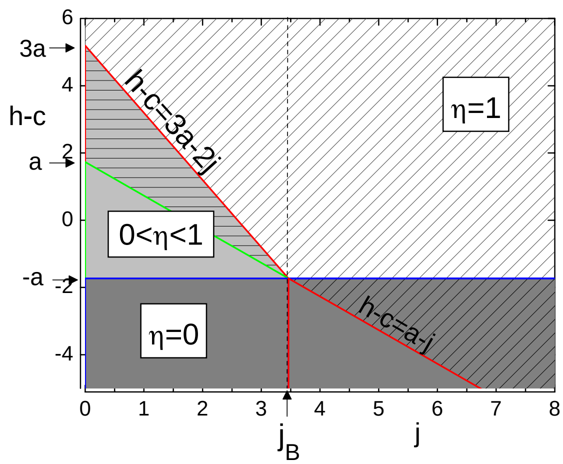

Figure 4: Phase diagram in the plane $(j, h-c)$: The region with diagonal stripes corresponds to optimal prices for which all the customers buy the product. For $j < j_B \equiv 2a$, the dark grey region corresponds to parameter values hindering adoption ($\eta = 0$). In the light grey region the fraction of buyers grows linearly with $h-c$, as can be seen in figure 3.In the horizontally hashed region, the monopolist's profit has two relative maxima, which become equal at the line $h-c = 3a-2j$. For larger values of $h-c$ there is a price ensuring full adoption. For $j > j_B$ and $h-c < a$, the penetration rate is a multivalued function of the price. Consequently, the monopolist has two possible strategies. In the present case of a uniform distribution, due to the discontinuities of $f(x)$, the solution with a small fraction of buyers is $\eta = 0$, and a high-price strategy cannot thus be implemented.

adoption ($\eta = 1$). Surprisingly, for $a-j < h-c < 3a-2j$, the optimal price is the highest one, for which $\eta < 1$. On the other hand, if $j > j_B$ the fraction of buyers is multivalued, and for $a-j < h-c < -a$ the monopolist has two solutions. However, due to the degeneracy of the uniform distribution, one of them (the one with the higher price) corresponds to $\eta = 0$; at the lower price there may be full adoption provided that the customer parameters satisfy $h-c > a-j$. Otherwise, $\eta = 0$. However, even if the inequality is satisfied, the question of whether the customers will actually buy or not is a coordination problem, whose issue depends on the dynamics of the adoption process.

The fraction of buyers, the optimal price and the corresponding monopolist's profit are represented as a function of $h-c$ on figures 3, for different values of j.

We determined the phase diagram in the case a smooth distribution, namely, the logistic [12]. It presents a richer structure, but with similar features as the uniform distribution analyzed here.

5 Dynamical features

We shortly discuss here some dynamical aspects, considering a market parallel dynamics with myopic customers: all the agents make thir decisions at time t based on the observation of the behaviour of the other agents at time $t-1$. The adoption by a single agent in the population (a "direct adopter") may then lead to a significant change in the whole population through a chain reaction of "indirect adopters" [8]. Within the McF model, the dynamics for the fraction of adopters in the large N limit is then given by

$$\eta(t) = 1 - F(p - h - j\,\eta(t-1)) \tag{25}$$

and $\eta(t)$ converges to a solution of the fixed point equation (11). As we have seen, given $j > j_B^u$, h and p, two stable and one unstable fixed points appear in (11) for values of $j \equiv J/\sigma$ large enough (large J or small σ). The stable solutions correspond to the two possible levels of η at a given price. Varying the price smoothly, a transition may be observed between these phases. The jump in the number of buyers occurs at different price values according

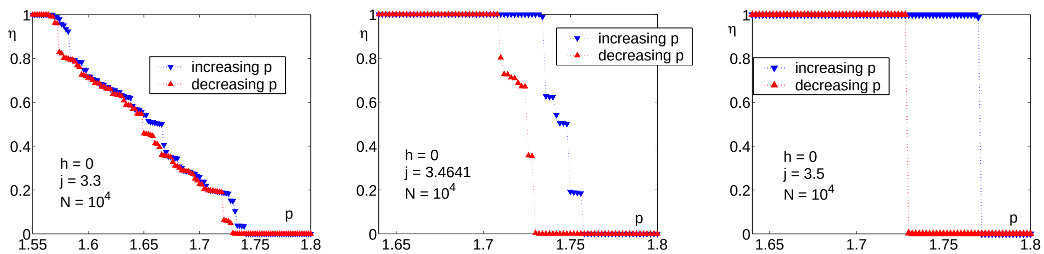

Figure 5: Behaviour of the demand as a function of the price, for social weight values at j_B^u and values slightly smaller and larger, showing that hysteretic effects in the case of finite size systems do not disappear precisely at $j = j_B^u$, in contrast with the theoretical predictions obtained in the mean field limit $N \to \infty$.

to whether the price increases or decreases, leading to *hysteresis loops*. In some cases, the number of customers evolves through a series of clustered flips (between $\omega_i = 1$ and $\omega_i = 0$), that we call avalanches (notice that these avalanches are dynamical features that arise during the updating process, and are not the avalanches referred to in [15], leading to metastable states. For small values of j there is a single fixed point for all values of P, and no hysteresis at all [8].

The curves in Figure 5, represent the fraction of customers η as a function of the price, obtained through a simulation of the whole demand system. The results correspond to upstream and downstream trajectories, obtained upon decreasing (increasing) the prices stepwise. We observe a *hysteresis* phenomenon with discontinuous transitions between the low and high adoption regimes. Typically, along the downstream trajectory (with increasing prices) the externality effect induces a strong resistance of the demand system hindering the number of customers from decreasing. In all these cases, avalanches at which large fractions of customers change their state occur at the so-called “first order phase transition”.

6 Conclusion

In this paper, we have compared two extreme special cases of discrete choice models, the McFadden (McF) and the Thurstone (TP) models, in which the individuals bear a local positive social influence on their willingness to pay, and have random heterogeneous idiosyncratic preferences. In the McF model the latter remain fixed, and give raise to a complex market organisation due to the corresponding quenched disorder: the McF model belongs to the class of Random Field Ising models (RFIM). In the TP model, all the agents share a homogeneous part of willingness to pay but have an additive, time varying, idiosyncratic characteristic. When the latter is drawn from a logistic distribution, this model corresponds to a finite temperature Ising model, that is, with an annealed (thermal) disorder. Thus, the McF and TP models, which are considered as equivalent in the economics literature, have quite different properties from the physicist's point of view. In this paper we have discussed some of them, and their consequences on the market's behaviour.

Considering that the monopolist optimises his own profit, we have exhibited a new first order phase transition: when the social influence is strong enough, there is a regime where, upon increasing the mean willingness to pay, or decreasing the production costs, the optimal monopolist's solution jumps from one with a high price and a small number of buyers, to one with a low price and a large number of buyers.

We illustrated the general conclusion on a simple example where the idiosyncratic part of the willingness to pay is uniformly distributed among the customers.

We have only considered fully connected systems: the theoretical analysis of systems with finite connectivity is more involved, and requires numerical simulations. The simplest configuration is one where each customer has only two neighbours, one on each side. The corresponding network, which has the topology of a ring, has been analysed numerically by Phan et al. [8]) who show that the optimal monopolist's price increases both with the degree of the connectivity graph and the range of the interactions (in particular, in the case of a small world). Buyers' clusters of different sizes may form, so that it is no longer possible to describe the externality with a single parameter, like in the mean field case. A more systematic study of these properties will be the subject of future work.

Acknowledgements

The authors acknowledge support from the ACI Complex Systems in Social and Human Sciences (CNRS and French Ministry of Education), project ELICCIR.

References

[1] Kirman A.P. The economy as an interactive system. In Lane Arthur, Durlauf, editor, *The Economy as an Evolving Complex System II*, pages 491–531. Santa Fe Institute Studies in the Sciences of Complexity, Volume XXVII, Addison-Wesley Pub.Co, (1997).

[2] Kirman A.P. The economy as an evolving network. *Journal of Evolutionary Economics*, **7**:339–353, (1997).

[3] Arthur W.B., Durlauf S.N., and Lane D.A. Eds. The economy as an evolving complex system ii. Reading Ma., (1997). Addison-Wesley Pub.Co.

[4] Durlauf S.N. Statistical mechanics approaches to socioeconomic behavior. In Lane Arthur, Durlauf, editor, *The Economy as an Evolving Complex System II.* Santa Fe Institute Studies in the Sciences of Complexity, Volume XVII, Addison-Wesley Pub.Co, (1997).

[5] Durlauf S.N. How can statistical mechanics contribute to social science? *Proceedings of the National Academy of Sciences*, **96**:10582–10584, (1999).

[6] Weisbuch G. and Stauffer D. Adjustment and social choice. *Physica A*, **323**:651–662, (2003).

[7] Phan D., Gordon M.B., and Nadal J-P. Statistical mechanics approaches in economics: an insight from statistical mechanics. In Nadal J-P. Bourgine P., editor, *Cognitive Economics.* Springer, (2003).

[8] Phan D., Pajot S., and Nadal J-P. The monopolist's market with discrete choices and network externality revisited: Small-worlds, phase transition and avalanches in an ace framework. In *Ninth annual meeting of the Society of Computational Economics, University of Washington, Seattle, USA, July 11-13, 2003*, (2003).

[9] Bulow J.I., Geanalopolos J.D., and Klemperer P.D. Multimarket oligopoly: strategic substitutes and complements. *Journal of Political Economy*, **93/3**:488–511, (1985).

[10] Anderson S.P., DePalma A., and Thisse J-F. *Discrete Choice Theory of Product Differentiation.* MIT Press, Cambridge MA, (1992).

[11] Thurstone L.L. Psychological analysis. *American Journal of Psychology*, 38:368–398, (1927).

[12] Nadal J-P., Phan D., Gordon M.B., and Vannimenus J. Multiple equilibria in a monopoly market with heterogeneous agents and externalities. *submitted to Quantitative Finance*, (2004).

[13] McFadden D. Econometric analysis of qualitative reponse models. In Griliches Intrilligator, editor, *Handbook of Econometrics, Vol II.* Elsevier Science B.V, Amsterdam, (1974).

[14] Tirole J. Fudenberg D. *Game Theory.* MIT Press, Cambridge, MA, (1993).

[15] Sethna J.P., Dahmen K., Kartha S., Krumhansl J.A., Roberts B.W., and Shore J.D. Hysteresis and hierarchies: Dynamics of disorder-driven first-order phase transformations. *Physical Review Letters*, **70**:3347–3350, (1993).