Emergence in multi-agent systems: cognitive hierarchy, detection, and complexity reduction part I: methodological issues

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1 Introduction

In a pioneering book on ”artificial society” and multi-agent simulations in social sciences, (Gilbert and Conte 1995) put the emphasis on ”emergence” as a key concept of such approach: ”Emergence is one of the most interesting issues to have been addressed by computer scientists over the past few years and has also been a matter of concern in a number of other disciplines, from biology to political science” (op.cit. p.8). More recently, Agent based Computational Economics (ACE) put the emphasis on the question of emergence, following for instance (Tesfatsion 2002a) or (Axtell and Epstein and Young 2001) The present paper provides a formal definition of emergence, operative in multi-agent framework designed by Agent Oriented Programming, and which makes sense from both a cognitive and an economics point of view. Starting with a discussion of the polysemous concept of emergence, the first part of this paper is dedicated to clarifying the question by focussing on the problem of modelling cognitive agents in artificial societies. The key questions are introduced by way of a paradigmatic example. The second part of this paper is dedicated to introducing and discussing operative definitions and related implications. In order to illustrate our formal definition of emergence, a companion paper (Phan and Galam and Dessalles, 2005) discusses the ACE population game model of (Axtell and Epstein and Young 2001) and builds a multi-level-model based on the formal framework introduced in this paper.

2 From emergentism to emergent behaviour in ACE model

In this section, we first discuss different definitions of emergence, and the related background. In order to focus on the problem of modelling cognitive agents in artificial society, we next considering a paradigmatic example, and briefly discuss Schelling’s model of spatial segregation (Schelling 1969, 1971, 1978), which is a pioneering study of an emerging social phenomenon in social science.

2.1 Emergence: one word, several meanings

The notion of emergence has several meanings. In the vernacular language, emergence denotes both a gradual beginning or coming forth, or a sudden uprising or appearance; to emerge also means to become visible, in example, emergence denotes the act of rising out of a fluid. This latter sense is close to its Latin roots, where *emergere* is the opposite of *mergere*: to be submerged. In the following, we relate the ”act of rising out” to the arising of some phenomenon in a process, and note the fact that to become visible presupposes some observer. In other words, the common sense of emergence is linked to the meaning of a process that produces some phenomenon that might be detected by an observer. In the field of science, emergence has been used by Newton in optics. By the 19th century the word ”emergent” is introduced into the fields of biology and philosophy. In the latter, Emergentism has a long history, from Mill’s chapter: ”Of the Composition of Causes” in *System of Logic* (1843) to the contemporary debates about the philosophy of mind, known as ”the mind - body problem”. For a synthesis, see among others: (McLaughlin, 1992, 1997; Van de Vijver, 1997; Emmeche, et al. 1997). Philosophical emergentism deals with questions of both reductionism and holism. (Lewes 1874) for instance places emergence at the interface between levels of organisation. For descriptive emergentism, the properties of the ”whole” cannot be defined by the properties of the parts, and results in part from some irreducible macro causal power.

In this debate around the definition of emergence, some authors have proposed to distinguish between different kinds of emergence, as for example ”nominal”, ”weak” and ”strong” emergence for (Bedau 1997,2002), or ”weak” ”ontological”, and ”strong” emergence for (Gillet 2002a-b). Both authors refer to debates about reductionism as well as about the so-called mind-body problem, discussing in particular the notion of Supervenience, introduced by (Davidson 1970, 1980) and discussed by (Kim 1992, 1993, 1995, 1999) from the point of view of emergence. As ”weak” emergence deals with upward causation and reductionism, Gillet and Bedeau relate ”*strong emergence*” to the question of ”*downward causation*” (Kim, 1992; Bedau, 2002) or ”macro-determinism”, widely advocated by (Sperry 1969, 1986, 1991) to deal with the mind-brain interactions, and by (Campbell 1974) to deal with hierarchically organized biological systems. According to strict downward causation, the behaviour of the parts (down) is determined by the behaviour of the whole (up). For instance, parts of the system may be restrained by some act in conformity with rules given at the system level. Causation would come ”downward” in conformity with a holist principle rather than upward, according to a reductionist principle.

In this paper, we do not address theses questions directly, as we limit ourselves to discussing social behaviours in artificial societies; but the opposition between *downward* versus *upward* causation proves to be a central one in the field of social sciences. According to (Granovetter 1985), the sociologist’s approach would be ”over socialized” (downward) while the economist’s approach would be ”under socialized” (upward/methodological individualism). Currently, both approaches have been sophisticated and are often mixed. The present paper is an attempt to integrate them in one single framework, in which the ’whole’ is a collective of agents (upward causation / methodological individualism), but the agents are in are to some extent constrained by the whole (downward causation), by way of the ”*social dimension*” of their belief as well as their perception of social phenomena. For the purpose of this paper, we rely on the distinction, proposed by (Muller 2002) in the field of multi-agent systems, between ”weak” and ”strong” emergence. The latter refers to a situation in which agents are able to witness the collective emergent phenomena in which they are involved, which opens the road for both upward and downward causation.

In Agent based Computational Economics, ”emergence” is strongly related to the Santa Fe approach to complexity (SFAC). In accordance with descriptive emergentism, SFAC calls ”emergence” the arising at the macro level of some patterns, structures and properties of a complex adaptive system that are not contained in the property of its parts. But conversely, emergence can often be explained by upward mechanisms. Interactions between parts of a dynamic system are the source of both complex dynamics and emergence. An interesting part of the emergence process concerns the forming of some collective ”order” (coherent structures or patterns at the macro level) as a result of agents' interactions within the system's dynamics, in the presence of a specific attractor. For the observer, this collective order makes sense by itself and opens up a radically new global interpretation, because it does not initially make sense as an attribute of the basic entities.

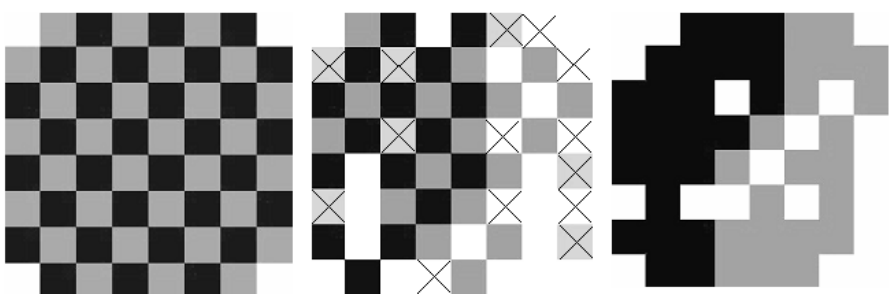
Formally, in multi-agent systems, emergence is a central property of dynamic systems based upon interacting autonomous entities (the agents). The knowledge of entities' attributes and rules is not sufficient to predict the behaviour of the whole system. Such a phenomenon results from the confrontation of the entities within a specific *structure of interaction*. That is, better knowledge of the generic properties of the interaction structures would make it easier to have better knowledge of the emergence process (ie. *morphogenetic dynamics*). From this point of view, to denote a phenomenon as ”emergent” does not mean that it is impossible to explain or to model the related phenomenon. For this reason (Epstein 1999) uses the word ”generative” instead of ”emergent” in order to avoid a philosophical debate about emergence.

Various attempts have been made to define emergence in an ”objective” way. Some definitions refer to self-organisation (Varela and Thompson and Rosch 1991), to entropy changes (Kauffman 1990), to non-linearity (Langton 1990), to deviations from predicted behaviour (Rosen 1985; Cariani 1991) or from symmetry (Palmer 1989). Other definitions are closely related to the concept of complexity (Bonabeau et al. 1995a, 1995b; Cariani 1991; Kampis 1991). In statistical physics (Galam, 2004), as well as for models in economics or social sciences explicitely based upon theses models (see for instance (Durlauf 1997, 2001) and the pioneering work of (Galam et al. 1982), emergence may be related with an *order parameter* which discriminates between at least two phases, each one with a different symmetry associated respectively to a zero and non-zero value of the order parameter. Each problem has its specific order parameter. For instance in the Ising model, where individual spins can takes the value $\{-1, +1\}$, the order parameter is the magnetization M, given by the sum of all the spin values divided by their total number. When $M = 0$, the state is paramagnetic, i.e. disordered in the spin orientations, while long range order appears as soon as $M \neq 0$. A majority of spins are then oriented along either -1 or $+1$, and an order is likely to emerge. Two ordered phases are thus possible in principle, but only one is effectively achieved. The order parameter provides a ”signature” for the emergent phenomenon. Although these definitions make use of concepts borrowed from physics and information science, they all involve inherently contingent aspects, as the presence of an external observer seems unavoidable. Even a change in entropy supposes that an observer be able to assess the probability of various states.

The *unavoidable presence of an observer* does not preclude, however, the possibility of extending the definition of emergence to include non-human observers or observers that are involved in the emerging phenomenon. In our quest for ”strong emergence”, we wish to assign the role of the observer to elements of the system itself, as when individuals become aware of phenomena affecting the whole society. This kind of self-observation is only possible because what is observed is a simplified state of the system. *Emergence deals* precisely *with simplification*.

2.2 What does emerge in Schelling’s model of spatial segregation?

Schelling’s model of spatial segregation (Schelling 1969, 1971, 1978) is a pioneering example of an emerging phenomenon resulting from social interaction. Schelling’s aim was to explain how segregationist residential structures could spontaneously occur, even when people are not so very segregationist themselves. The absence of a global notion of segregationist structures (like the notion of ghettos) in the agent’s attributes (preferences) is a crucial feature of this model. Agents do not choose between living or not living in a segregationist structure, but have only local preferences concerning their preferred neighbourhood. Moreover, people have only weak segregationist behaviour, but the play of interactions generates global segregation. In Schelling’s original model, agents were placed on a 8-by-8 chessboard as shown in Figure **??** (Java applet).



(1-a) fully integrated population equilibrium (1-b) discontented agents are crossed (1-c) convergence after 4 iterations
Source: http://perso.univ-rennes1.fr/denis.phan/complexe/schelling.html and (Phan 2004

Fig. 1. Original (checkerboard) Schelling Model

Taking the “colour“ of agents as criterion for discrimination, agents choose a location where to live, depending on their individual tolerance threshold of different colours in their neighbourhood. Agents interact only locally with their 8 direct neighbours (within a so-called ”Moore” Neighbourhood). No global representation about the residential structure is available to them. Though agents may be weakly segregationist (each agent would stay in a neighbourhood with up to 62.5% of people with another colour), segregation occurs. Schelling used the following rule: an agent with one or two neighbours will try to move unless one of the two neighbours has the same colour as its own (which means a local tolerance of 50%); an agent with three to five neighbours requires at least two agents of same colour to stay (that is 33%, 50% and 60% local tolerance), and one with six to eight neighbours will stay if at least three of them are of the same colour (50%, 57,1%, 62,5% local tolerance).

Under Schelling’s behavioural assumption, a ”fully integrated structure” (Figure **??**-a) is an equilibrium (an order) because no agent wishes to move. A ”fully integrated structure” is a structural pattern in which agents’ colours alternate in all directions. Because of border effects, no agent is located in the corners. The ”fully integrated structure” is an unstable equilibrium. A slight perturbation is sufficient to induce a *chain reaction* and the emergence of local segregationist patterns. In his example, Schelling extracted twenty agents at random, and added five at random in the free spaces. The discontented agents (crossed in Figure **??**-b) move at random towards a new location in agreement with their preferences. These moves generate new discontented agents by a chain reaction until a new equilibrium is reached. In such equilibrium, local segregationist patterns appear, like in Figure **??**-c.

Local interactions are sufficient for spatial homogeneous patterns to occur; spatial segregation is an emerging property of the system’s dynamics, while not being an attribute of the individual agents. Sometimes, local integrated (non-homogeneous) patterns may survive in some niches. But such integrated structures are easily perturbed by random changes, while homogeneous structures are more stable (frozen zones). Complementary theoretical developments on Schelling’s model of segregation can be found in the growing literature on this subject, for instance among economists like (Zhang 2004a-b), (Pancs, Vriend 2003, 2004), or sociologists like (Broch Mare 2004). Examples of advances in empirical investigations can be found in (Clark 1991), (Sethi Somanathan 2001), (Koeler, Svoretz 2002), and experimentations in Ruoff, (Schneider 2004). Our aim in this paper is to address emergent phenomena, as instantiated in Schelling’s model, in a new way. Emergence is currently debated for its cognitive and sociological aspects, from ontological and epistemic perspectives, in relation with the modern philosophy of mind (for a selection of papers, see for instance (Intellectica 1997), and (Gilbert 1995), for links with sociology. There is also a debate within the artificial intelligence, artificial life and artificial society fields see (Gilbert,Conte,1995) in this later field. Entering or even summarizing those debates would fall outside the scope of the present paper. But some fundamental questions are worth asking about knowing in what way emergence occurs in Shelling’s model. Who is the observer? What does the higher level of organisation consist in? For whom does this level make sense? (Figure **??**)

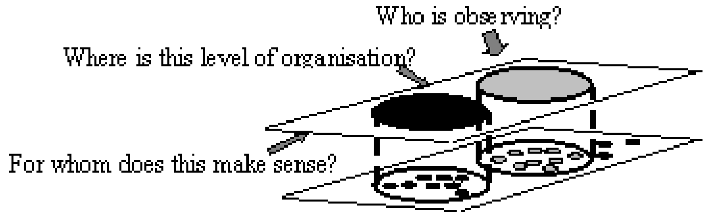


Fig. 2. Questions of emergence

In various definitions of emergence, the presence of an external observer seems unavoidable. Levels of organisation depend on an observer being able to discern subparts in the system and appropriate relations between them. There is no consistent way to say that some new phenomenon occurs at a higher level, be it some new form of operational closure or any form of deviation from expected behaviour, unless there was some pre-existing way to observe that higher level.

3 Emergence in ACE : from case studies to a formal definition

The first and second subsections provide two definitions coherent both with the design of multi-agent systems used in Agent Based Computational economics (Tesfatsion 2002; Phan, 2004) and with important related features, like cognitive hierarchy, detection, and complexity. The first one from (Bonabeau, Dessalles 1997) defines the emergence as an unexpected complexity drop in the description of the system by a certain type of observer. The second one from (Muller 2002) defines emergence as a phenomenon observed at the interface of description levels. The latter definition introduces a useful distinction between ”weak” and ”strong” emergence.

3.1 Emergence as a complexity drop

In (Bonabeau, Dessalles 1997), emergence is defined as an unexpected complexity drop in the description of the system by a certain type of observer. Such a definition is claimed to subsume previous definitions of emergence, both structural (dealing with levels of organisation) and epistemological (dealing with deviation from some model's predictions). In each case, the observer is able to detect a structure, such as the presence of relations holding between parts of the system, or some form of behaviour like a characteristic trajectory. Structural emergence occurs whenever the system turns out to be more structured than anticipated. This augmentation of structure can be characterised by a decrease of complexity.

$$E = C_{exp} - C_{obs}$$

Here, E stands for the amplitude of the emergence, C_{exp} is the expected structural complexity and C_{obs} the structural complexity actually observed. Structural complexity is defined as the algorithmic complexity relative to a given set of structural descriptors.

Algorithmic complexity, as defined by Kolmogorov, Chaitin and Solomonov, is defined by the shortest description that can be given of the system using a Turing Machine (Li, Vitanyi 1993). This definition is sometimes considered of little use for finite systems, as the set of all systems of same size can be ordered; since each of these system can be characterised by its rank, nothing prevents the actual system to appear as the simplest one if it happens to be number one. In order to use algorithmic complexity to describe finite system, we abandon the generality of Turing machines, considering that the description tools are imposed by the observer. We define the relative algorithmic complexity (RAC) of a system as the complexity of the shortest description that a given observer can give of the system, relative to the description tools available to that observer. Emergence occurs when RAC abruptly drops down by a significant amount.

For our purpose here, we must restrict the definition. We consider a specific class of observers, in order to get closer to what human observers would consider as emergence. Following (Leyton 2001), we impose the observer's description tools to be structured as mathematical groups. In other words, any level of organisation that can be observed has operational closure and is structured as a group, and the only structures that can be observed are the invariant of a group of operations. Moreover, the observer is supposed to have hierarchical detection capabilities. This means that all elements of the system that the observer can consider have themselves a group structure. The observer may be considered as being a 'Leyton machine', for which any structure is obtained through a group-transfer of other structures (Leyton 2001).

Let us illustrate how emergence results from a complexity drop in Schelling's model. In a first stage, the external observer reconstructs the system by transferring (in the Leyton sense) one abstract inhabitant to form the entire population. The transfer group, in this case, is the group of 2-D translations. The operation is costly in terms of complexity, as each individual translation has to be instantiated. Then each abstract inhabitant is assigned a colour. This latter operation can be achieved through a transfer by the binary group Z/2Z. In a second stage, the external observer is now able to detect homogeneous clusters. She reconstructs the system in a different way. One first abstract cluster is obtained by translating one abstract inhabitant, as previously. Then this first cluster is itself translated to give the whole set of clusters. Finally, clusters are assigned colours through the binary group. Emergence, in this example, comes from the fact that the second construct is significantly simpler than the first one. The reason is that there are less colour assignments: only one per cluster instead as one per inhabitant. A crucial requirement for the emergence to be noticeable is that the shape of clusters be simple. For the system to be fully instantiated, the second construct must reshape the limits of each cluster through various groups of geometrical transformations. If there were no colours, or if the clusters had random shapes, there would be no gain in complexity. Conversely, emergence would be maximum in the extreme case in which all clusters had identical shapes, e.g. if they were square blocks.

Each transfer group can be seen as an organisation level. In Schelling's model, there could be more levels of organisation, for instance if clusters were arranged in a chessboard-like pattern. The Leytonian construct would be different and less complex in this case: the first cluster would be assigned a colour, then it would be duplicated through a binary symmetry group operating in colour space, then the couple would be transferred through the group of integer translations of the plan.

For structural emergence to occur, it is important that there be an unexpected complexity decrease. This may happen either because the higher structure detection was delayed, as when you take time to recognise a Dalmatian dog in a pattern of black and white spots. It may also happen when adding a new observable, instead of increasing the overall complexity of the system for the observer, paradoxically deceases it (Bonabeau, Dessalles, 1997). This latter case is well illustrated by our extension of Axtell et al.'s experiment (see Phan, Galam, Dessalles, 2005).

3.2 Emergence occurring in a system with several levels

Following Forrest's definition of emergent calculation (Forrest 1990), (Müller 2002) defines emergence in SMA as occurring between two organisation levels, distinguishing the process and the observation of that process. The process concerns the evolution of a system formed by entities in interactions. These interactions may generate *observable epiphenomena*. At the observation level, *epiphenomena are interpreted as emerging through specific calculation* (i.e. like order parameter). For Müller, ”*weak emergence*” arises when the observer is external to the system, while ”*strong emergence*” arises when the agents involved in the emerging phenomenon are able to perceive it. In this later configuration the identification of epiphenomena by the agents in interaction in the system will involve a *feedback from the observation to the process*. There is a coupling between the process level and the observation level by the way of the agents. Emergence is thus *immanent* in such a system.

More specifically, for Müller, a phenomenon is emergent if:

- (A) There is a system composed of agents in interaction with each other and with their environment. The description of this system as a process is formalized in a language D
- (B) The dynamics of this system produces a structural phenomenon observable in the ”traces of execution”
- (C) The global phenomenon is observed by an external observer (weak emergence) or by the agents themselves (strong emergence) and is described in a language distinct from D.

When compared with Forrest's definition (Forrest 1990), Müller's definition presupposes the existence of two languages of description, which are distinct according to the level considered. This distinction only materializes the presence of levels already hypothesised by Forrest. On the other hand, it is interesting to note that Müller distinguishes the system formed by the interacting agents from the process that governs their behaviour. This enables him to choose the position of the level of observation with respect to the agents. Müller's contribution lies then mainly in the distinction between two categories of emergence according to the position of the level of observation w.r.t. process. In strong emergence, agents are observers themselves, this de facto entails a feedback loop between the micro (agent based) level of observation and the macro level of the process. In weak emergence, the observer is external with the process and there is no necessarily coupling.

Müller illustrates weak emergence by means of the example of foraging ants which move between their nest and a food source. Each ant deposits on its passage some traces of pheromone which attract the other ants, and create an interaction between them (1). These interactions build a stable and observable phenomenon (2). An external observer may interpret this phenomenon as a ”path”. Moreover, the accumulation phenomenon based on interaction drives the ant colony to find the shortest path between their nest and a source of food. Emergence is weak because the dynamics depends only on the traces of pheromone (1-2) and not on the qualification of these traces as an ”shortest or optimal path”, which does not exist in the ants' head.

The category of strong emergence is important for to model artificial societies (Gilbert 1995). Indeed, the reflexivity meditated by the agents' ”consciousness” appears to be a determinant characteristic that distinguishes systems involving human agents from systems made of non conscious or material entities.

In Schelling's model, there would be strong emergence if agents, rather than merely sampling neighbouring densities, were able to perceive forming homogeneous clusters in the town and if their perception could affect their decisions. Strong emergence is particularly important in economic modelling, because the behaviour of agents may be recursively influenced by their perception of emerging properties. Emerging phenomena in a population of agents are expected to be richer and more complex when agents have enough cognitive abilities to perceive the emergent patterns. Such feedback loops between emerging collective patterns and their cognitive components clearly occur among agents in human societies. They may obey laws that are still to be understood. Our aim here is to design a minimal setting in which this kind of strong emergence unambiguously takes place.

4 Conclusion

To summarize, if there is *strong emergence* in the sense of Müller, the system becomes reflexive, through the mediation of the agents. (A) Agents are equipped with the capacity to observe and to identify an epiphenomenon in the process which represents the evolution of the system in which they interact. This capacity of observation and the field of such observation must then be sufficiently broad to encompass the phenomenon as a global one. (B) The agents can describe this epiphenomenon in a ”language” other than that which is used to describe the process (C) The identification of an ”emergent” epiphenomenon by the agents involves a change of behaviour, therefore a feedback of the level of observation on the process

Emergent phenomena are naturally described in a two-level architecture (Figure 3). In such a framework, objects at the two levels only exist because some observer is able to detect them. The detected object at the upper level is composed by objects of the first level. Correspondingly, the upper level detector is triggered by the activity of lower level detectors. The system's complexity, defined as the minimal description that can be given of its state, drops down by a significant amount when an upper-level

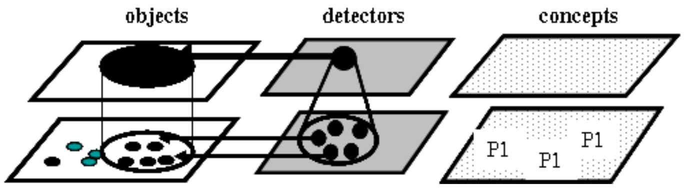


Fig. 3. Parallelism between hierarchies : description, observations and conceptual level

detector becomes active, as its activity subsumes the activity of several lower-level detectors.

According to this point of view, one can reinterpret the Müller's definition using a distinction due to Searle (1995) between entities that are independent of the observer (the process and the phenomena which results from it) and entities that occur within the observer (identification and interpretation of an epiphenomenon). According to this interpretation, *emergence becomes a category relative to an observer*, and in the case of a human observer (or an agent supposed to be represented an human), a subjective category. Note that Müller's definitions and the above definition of structural emergence as complexity drop are compatible. Müller's distinction between two description languages presupposes that the upper language, available to the observer, provides it with a simpler description of the epiphenomena than what was available at the process level.

In order to illustrate our formal definition of emergence, a companion paper (Phan, Galam, Dessalles 2005) discusses the ACE model of (Axtell and Epstein and Young 2001) and builds an extension based on the formal framework introduced in this paper. In the basic model, agents tend to correlate their partners' behaviour with fortuitous visible but meaningless characteristics (tags). On some occasions, these fortuitous tags turn out to be reliable indicators of dominant and submissive behaviour in an iterative Nash bargaining tournament. One limit of this model is that dominant and submissive classes remain implicit within the system. Classes only emerge in the eye of external observers (weak emergence). We enhance the model to allow for strong emergence. In a two-level framework, agents get an explicit representation of the dominant class whenever that class emerges, thus implementing strong emergence.

References

1. Axtell R. Epstein J.M., Young H.P. (2001) The Emergence of Classes in a Multi-agent Bargaining Model in Durlauf, Young eds.,Social dynamics, The MIT Press, p. 191-212
2. Bedau, M.A. (1997) Weak Emergence, Nos, Vol. 31, Supplement: Philosophical Perspectives, 11, Mind, Causation, and World, p. 375-399
3. Bedau, M. A.((2002) Downward causation and the autonomy of weak emergence, Principia 6-1 June, special issue on Emergence and Downward Causation, p. 5-50
4. Bonabeau E., Dessalles J-L. (1997) Detection and Emergence, Intellectica, 25:89-94.
5. Bonabeau, E., Dessalles, J.-L. and Grumbach, A. (1995a) Characterizing emergent phenomena (1): A critical review. Rev. Int. Syst. 9:327-346.

6. Bonabeau, E., Dessalles, J.-L. and Grumbach, A. (1995b) Characterizing emergent phenomena (2): a conceptual framework. Rev. Int. Syst. 9:347-369.
7. Broch, Mare (2003) Neighborhood Choice and Neighborhood Change California Center for Population Research On-Line Working Paper Series CCPR-003-03, June.
8. Campbell DT (1974), Downward causation, in hierarchically organized biological systems. In: Studies in the Philosophy of Biology: Reduction and Related Problems, Ayala, Dobzhansky, eds. Macmillan, London
9. Cariani, P. (1991) Adaptivity and emergence in organisms and devices, World Futures 31, pp. 49-70, Gordon & Breach Science Publishers
10. Clark W.A.V. (1991) Residential Preferences and Neighborhood Racial Segregation: A Test of the Schelling Segregation Model, Demography 28, P. 1-19.
11. Davidson, D. (1970) Mental Events, in Lawrence Foster and J. W. Swanson, eds., Experience and Theory, Duckworth, London.
12. Davidson D. (1980) Actions and Events, Clarendon, Oxford
13. Durlauf S.N. (1997) Statistical Mechanics Approaches to Socioeconomic Behavior, in Arthur, Durlauf, Lane (eds), *The Economy as an Evolving Complex System II*, Santa Fe Institute Studies in the Sciences of Complexity, Volume XXVII, Addison-Wesley.
14. Durlauf.S.N. (2001) A Framework for the Study of Individual Behavior and Social Interactions, Sociological Methodology Volume 31 Issue 1, 1 January pp. 47-87
15. Emmeche, C., Kppe, S. & Stjernfelt, F. 1997. Explaining emergence-towards an ontology of levels, Journal for General Philosophy of Science, 28: 83-119.
16. Epstein J.M. (1999) Agent-Based Computational Models and Generative Social Science, Complexity, Vol.4-5:41-60.
17. Ferber, J., Labbani O., Müller J.-P., Bourjault. A. (1997) Formalizing emergent collective behaviours: Preliminary report. International Workshop on Decentralized Intelligent and Multi-Agent Systems, St. Petersburg, p. 113-122.
18. Forrest S. (1990) Emergent Computation : Self-organizing, Collective, and Cooperative Phenomena in Natural and Artificial Computing Networks, in Emergent computation, MIT Press, Cambridge, M.A., 1990, p. 1-11.
19. Galam S., Gefen Y. , ShapirY. (1982) Sociophysics: a new approach of sociological collective behavior, Journal of Mathematical Sociology **9**, 1
20. Gilbert N. (1995) Emergence in social simulations, in Gilbert, Conte. eds. 114-156.
21. Gilbert N., Conte R. eds. (1995) Artificial Societies : The Computer Simulation of Social Life UCL Press London.
22. Gillett C. (2002a) The Varieties of Emergence: Their Purposes, Obligations and Importance, Grazer Philosophische Studien, vol.65, pp.89-115.
23. Gillett C. (2002b) Strong Emergence as a Defense of Non-Reductive Physicalism: A Physicalist Metaphysics for 'Downward' Determination, Principia 6-1 June, special issue on Emergence and Downward Causation,, p.83-114.
24. Granovetter M.S. (1985) Economic Action and Social Structure: The problem of embeddedness, American Journal of Sociology, 91 p. 481-510.
25. Kampis, G. (1991) Self-Modifying Systems in Biology and Cognitive Science Pergamon
26. Kim, J. (1992) Downward causation, in emergentism and nonreductive materialism, in Beckermann, Flohr, Kim, eds., Emergence or Reduction? Walter de Gruyter.
27. Kim, J. (1993) Supervenience and Mind, Cambridge: Cambridge University Press.
28. Kim, J. (1995) Emergent properties, in T. Honderich, ed. The Oxford Companion to Philosophy, Oxford University Press p. 224.
29. Kim, J. (1999) Making sense of Emergence, Philosophical Studies, 95 p.3-36.
30. Koehler G. Skvoretz J. (2002) Preferences and Residential Segregation: A Case Study of University Housing WP

31. Langton C. (1989) Computations at the edge of chaos, Physica D, 42
32. Li M. Vitanyi P.M.B. (1993) An Introduction to Kolmogorov Complexity and Its Applications. Springer-Verlag, New York, Berlin
33. Lewes, G. H. (1874) in Emergence, Dictionnaire de la langue philosophique, Foulqui.
34. Leyton, M. (2001) A generative theory of shape, New York: Springer Verlag
35. McLaughlin B. P. (1992) The rise and fall of British emergentism, in Beckermann, Flohr, Kim, eds., Emergence or Reduction? Berlin: Walter de Gruyter.
36. McLaughlin B. P. (1997) Emergence and supervenience, Intellectica 25 p.25-43.
37. Mill, J.S. (1843) System of Logic, Longmans, Green, Reader, and Dyer, London.
38. Muller J.P. (2002) Des systèmes autonomes aux systèmes multi-agents : interactions, émergence et systèmes complexes Rapport de l'HDR, 8 novembre, 58p.
39. Palmer, R. (1989) Broken Ergodicity, in Stein ed. Lectures in the Sciences of Complexity, Vol.1, SFI Studies in the Sciences of Complexity, Addison-Wesley, Reading, Mass.
40. Pancs, R., Vriend, N.J. (2003) Schelling's Spatial Proximity Model of Segregation Revisited, Dept. of Economics Working Paper No. 487 Queen Mary, University of London
41. Phan D. (2004) From Agent-Based Computational Economics towards Cognitive Economics, in Bourgine, Nadal, eds., Cognitive Economics, Spinger Verlag p. 371-398.
42. Phan D., Galam S., Dessalles J.L. (2005) Emergence in multi-agent systems: cognitive hierarchy, detection, and complexity reduction part II: Axtell, Epstein and Young's model of emergence revisited, CEF 2005,11th International Conference on Computing in Economics and Finance, June 23-25, 2005, Washington, DC
43. Rosen, R. (1978) Fundamentals of measurement and representation of natural systems, North Holland.
44. Ruoff G. Schneider G (2004) Segregation in the Class Room: An Empirical Test of the Schelling Model, meeting of the working group on decision theory of the German Political Science Association, University of Bamberg, July 9-10
45. Schelling T.S. (1969) Models of Segregation, American Economic Review, Papers and Proceedings, 59-2:488-493.
46. Schelling T. S.(1971) Dynamic Models of Segregation, Journal of Mathematical Sociology; 1, p.143-186.
47. Schelling T.S. (1978) Micromotives and Macrobehavior W.W. Norton and Co, N.Y.
48. Searle J.R. (1995) The Construction of Social Reality, Free Press
49. Sethi R. Somanathan R (2004), Inequality and Segregation, Journal of PoliticalEconomy p.112-6
50. Sperry, R. W.(1969) A modified concept of consciousness, Psychol. Rev. 76, 532-536
51. Sperry, R. W. (1986) Discussion: Macro- versus micro-determinism (A response to Klee), Philosophy of Science, 53(2), p.265-270.
52. Sperry, R. W. (1991) In defense of mentalism and emergent interaction, Journal of Mind and Behavior 12, 221-245
53. Tesfatsion L. (2002) Economic Agents and Markets as Emergent Phenomena, Proceedings of the National Academy of Sciences U.S.A., Vol. 99-S3:7191-7192
54. Tesfatsion L. (2002b) Agent-Based Computational Economics: Growing Economies from the Bottom Up, Artificial Life, Volume 8 - 1, MIT Press, p.55-82.
55. Van de Vijver G. (1997) Emergence et explication, Intellectica 25:7-23.
56. Varela F., Thompson E., Rosch E. (1991) The embodied mind, MIT Press.
57. Zhang J. (2004a) Residential Segregation in an All-Integrationist World, Journal of Economic Behaviour & Organization, 54, p.533,550
58. Zhang J. (2004b) A Dynamic Model of Residential Segregation, Journal of Mathematical Sociology, 28, p.147-170