Hierarchy of cognitive interactive agents and statistical mechanics : how Object Oriented Programming highlights the connection

Denis Phan
Leibniz-IMAG, ENST de Bretagne,
& ICI-(Université de Bretagne Occidentale)
e-mail : denis.phan@enst-bretagne.fr

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ABSTRACT

This paper underlines how Object Oriented Programming (OOP) used within an agent-based framework allows one to make explicit in a specific design pattern the cognitive hierarchy between types of agents and makes a family of a model tractable both for programming and for semantic purposes. We also discuss the transposition of tools, methods and concepts from one academic field to another, taking as an example the implementation of models with statistical mechanics features by economists. Based on the preliminary implementation of some learning rules, we argue that the agent-based framework and object oriented programming can help clarify the epistemic debate on the articulation between disciplines, as well as between levels of abstraction.

INTRODUCTION

Agent-based Computational Economics (ACE) is a computational approach to the modelling of economics. Tesfatsion (2001) defines ACE as "the computational study of economies modelled as evolving systems of autonomous interacting agents". "Starting from initial conditions, specified by the modeller, the computational economy evolves over time as *its constituent agents repeatedly interact with each other and learn from these interactions*". The resulting artificial economy may be viewed as a complex adaptive system (Weisbuch, 1991). Within the field of complex adaptive systems, economics and social systems must have specific features such as communication and cognition (Anderson *et al*.1988; Arthur *et al* 1997). More specifically, human agents, by way of subjective reasoning, have the capacity to reflect on themselves, as well as on their environments. Human agents learn from others through communication, by understanding, for instance an inter-subjective confrontation of their beliefs. Moreover, communication itself is a social and cognitive process that uses specific institutions like language. As a consequence, economic and social systems can be viewed as cognitive (complex adaptive) systems, like the agents themselves.

This paper focuses on *cognitive hierarchy between agents*, and underlines how Object Oriented Programming (OOP) used within an agent-based framework can help to make this hierarchy explicit and tractable both for programming and for semantic purposes through a specific design pattern. Links with other patterns, like the decreasing abstraction principle or the family of models are underlined. We argue that both agent-based models and OOP can contribute to clarifying the epistemic debate on the articulation between disciplines as well as between levels of abstraction.

In order to illustrate this question, we take as an example the use of results from statistical mechanics models for modelling economic systems with social interactions. This approach took off in the 90's (see Durlauf 1997; 1999 Brock, Durlauf 2001, Durlauf, Blume 2002). Here, we discuss simple examples taken from Gordon (2004) for statistical mechanics and Phan *et al*., (2004) for applications to economics. The first section of the paper presents the question of cognitive hierarchy between agents both from a conceptual and an OOP point of view. The second section briefly summarises the statistical mechanics reference model, then introduces a very simple economic model of discrete choices with social interactions, and finally highlights links with game theory. The last section presents illustrations obtained by way of preliminary investigations using Moduleco (Phan, Beugnard 2001; Phan 2004), an agent-based framework used to implement the resulting family of models.

I - COGNITIVE HIERARCHY BETWEEN AGENT AND OOP MODELLING OF EPISTEMIC LEVELS.

Cognitive hierarchy

To model economic and social systems as cognitive systems "from the bottom up", with artificial agents which behave in some way like human agents, it is interesting to identify explicitly the level of cognitive functions that agents are assumed to have. For the implementation, the multi-agent framework and OOP allow us to introduce agents' heterogeneity with respect to their cognitive capacities in a natural way. If all agents have the same cognitive capacity, such a distinction allows us to identify the level of cognition related to the model. In a multi-agent framework, it is also possible to have co-evolving agents with different cognitive capacities (Bourgine 1993), but this case lies beyond the field of the present paper. In economics, it is possible to classify models according to the degree of cognitive activity that agents are assumed to have. For instance, in *game theoretic models*, Walliser (1998) identifies four kinds of dynamic processes that may converge towards an equilibrium. These processes also have an order, since they require increasing cognitive capacities of the agents.

(1) In (the current interpretation of) an *evolutionary process*, each agent has a fixed strategy, and the reproduction rate of the strategy in the population is proportional to the payoff obtained through the agents

interactions. As in Darwinian processes, learning occurs at the population level only.

(2) In *behavioural learning*, each player modifies his strategies according to the observed payoff obtained from his past actions.
(3) In *epistemic learning*, each player updates his beliefs about others' future actions, according to their actions observed.
(4) In an *eductive process*, each player has enough information to perfectly simulate others' behaviour and immediately reaches equilibrium.

The last case corresponds to the "perfect information case" with (fully) rational agents in economic and game theory. In particular, "classical" non-cooperative game theory requires the "*common knowledge*" assumption which leads to a recursive cross-knowledge about the rules of the game. A rule R is "common knowledge" if all players know R, all players know that all players know R, all players know that all players know that all players know R, and so on. Because agents know everything, there is generally no learning at all, and the *ex post* realisation of agents' plans corresponds generally to the result of their rational computations *ex ante*. Because this case has no cognitive dimension, we do not take it into account, and focus on other cases.

It is interesting to note that such a hierarchy in the representation of human cognition capabilities corresponds in some way to the perceptible hierarchy in the living domain. For instance, Dennett (1996) represents the hierarchy of cognitive capacity in the phylogeny of living creatures by his "Tower of Generate-and-Test". At each phylogenetic stage, a qualitatively new cognitive capacity comes to enhance the existing ones, inherited from lower-level stages. At the lowest stage, he places *Darwinian* creatures, that have a rigid phenotype. At the second stage, *Skinnerian* creatures have a phenotype adaptable through reinforcement-learning capabilities. At the third stage, *Popperian* creatures have some capability to pre-select actions, given the available information coming from inheritance and/or acquisition. At the last stage, *Gregorian* creatures can enhance their individual performances through the use of "tools". Among the tools, language and models are special kinds of mental (symbolic) tools, and very important for cultural transmission. Last but not least, Conte (1999) introduces the notion of "*social intelligence*" as a property of *socially situated* agents. Such agents are subject to a double level hierarchy both in the social and cognitive dimension. Talking into account the cross-feedback between the social and cognitive dimension of agents at the first level, the second level in the cognitive and social dimension is viewed as an emergent phenomenon grounded in first level interactions. Like communication, this complex social organisation is temporarily avoided in this preliminary work. In a companion paper (Dessalles, Phan 2004), centred upon the question of social "emergence", where we attempt to formalise such cognitive / social hierarchies as a coupled system of (cognitive) levels with detection. In this first essay, levels of organisation in class hierarchy result from the agent / observer's perceptive structure which is organised as a detection hierarchy, in accordance with both Bonabeau, Dessalles (1997) and Müller (2000). This latter model must be viewed as a more sophisticated declension of cognitive hierarchy, implemented within the generic pattern proposed in this paper.

Given these different examples of classification, we introduce in this paper three cognitive levels coherent with game theoretic models, but likely to be interpreted in Dennett's way. More specifically, we characterise each level by the required level of *cognitive rationality*, following a distinction proposed by Walliser (1989) between "instrumental" and "cognitive" rationality in economic models.

(1) A *Reactive Agent* (RA) has a *pre-determined response* for a given state of his environment. Such an agent may represent both a material entity, such as a "spin" in a ferromagnetic set, or all living entities (Darwinian creatures) in their pre-determined behaviour. The level of cognitive rationality is null.
(2) A *Behavioural Agent* (BA) may modify his behaviour for a given state of his environment according to the observed (historic) payoff obtained from his past actions, following Skinnerian reinforcement learning. His level of cognitive rationality is low.
(3) An *Epistemic Agent* (EA) uses a model of his environment to pre-select actions. His level of cognitive rationality may be medium or high, depending on the scope of his belief, and on the sophistication of his cognitive tools. So, both Popperian and Gregorian levels are different levels of epistemic agents.

Both *Behavioural* and *Epistemic* agents have a specific status with respect to observation/cognition. Both are observers of the process in which they take part. A *behavioural observer* only takes into account the *visible characteristics* of his environment that are known to have an effect on some personal criteria (viability, hedonic index, etc.). An *epistemic observer* models this process in some way and can simulate it, by way of symbolic tools. Let us note that, from this point of view, an external observer (the modeller or the experimentalist) has some features in common with an epistemic agent. In particular, the OOP multi-agent framework with epistemic agent may share some components in order to design the experimentalist's point of view and the epistemic agent's point of view.

Let us note that it is sometimes possible to model the same behaviour at different levels of the cognitive hierarchy, using special assumptions about specific parameters or cognitive patterns. In this special case, if the model can be formalised in the classical mathematical way, the same model applies in all cases, and only the interpretation differs. However, in more general cases, such a reduction does not work: it is typically the field of interest to use a multi-agent framework with a cognitive hierarchy. Let us note that cases 2 and 3 are both qualified as "behavioural" by some authors like Camerer (1997). A given learning rule may be attached to one category or another, depending on the criterion retained. As a result, the frontier between such categories sometimes remains unclear in the literature, because some mathematical expression may be subject to non-univocal interpretations, depending on the point of view. One interest of OOP patterns is to make more explicit links between software Object ontologies and the epistemic and ontological dimensions of related categories both from a cognitive and a social point of view.

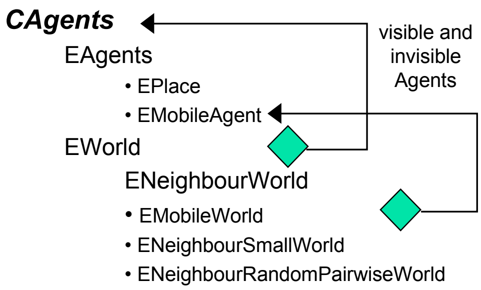
OOP architecture and hierarchy of class in Moduleco

A growing part of ACE uses "*computational laboratories*" i.e. a *multi-agent framework*, defining the structure of many possible applications, based on OOP. In order to *think by analogy*, OOP is very intuitive, and *useful for semantic purposes*. In this way, some parts of the software architecture may be closer to the conceptual organisation of the problem than to a specific procedural way of solving such a problem. Moreover, OOP provides a good conceptual environment to view the different levels of abstraction related to a specific problem, and to handle interdisciplinary analogies. An important feature of OOP is inheritance, meaning that classes are organised in a hierarchical way based on their degree of abstraction. Subordinate classes inherit the methods, dynamic links and other attributes of higher classes, but include additional ones or replace the higher classes' methods and attributes with more specialised substitutes.

The conceptual model of Moduleco relies on a CAgent class which is the root of the agents' hierarchy. The CAgent class, (an interface) defines only a list of methods necessary to be an Agent. Each agent makes a computation, "compute(), and validates this computation by changing his "state" (selected variables) by a "commit()" method, driven by a central scheduler. The direct subclasses are Eagent(s), which represents all individual agents, and EWorld that represents all *sets of agents*, but can be considered also as a single "composed" agent. In that way, an EWorld can be a set of EWorld(s). This recursive property may be useful in the case where we want to model a pyramid of hierarchical organisations or in the case of an "eductive" agent that would ontologically define his own representation of the world where he evolves (including himself).

Specialised EWorld(s) are ENeighbour-World(s) (Agents interconnected via a Neighbourhood), EMobileWorld which are composed of EPlaces, and EMobileAgents that move from EPlace to EPlace.

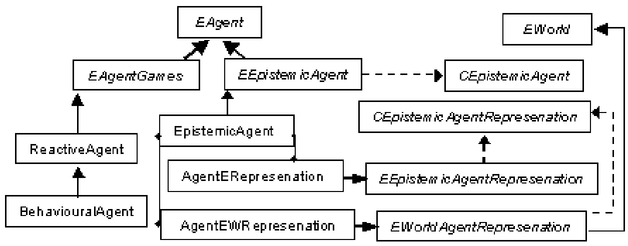
Figure 1 : Abstract agents in Moduleco



Design patterns have been widely used in software engineering in particular to provide highly abstract structures in OOP. A function of such patterns is to make the imitation process in programming easier and more efficient. In Multi-agent framework design, some patterns focus on agent design and inter-agents levels of organisation (Ferber, Gutknecht 1998; Gutknecht *et al.* 2001). In a multi-modelling perspective, Amblard *et al.* (2001) focus on organisation by "collection of models" according to the decreasing abstraction methodology (Lindenberg 1992). These authors suggest beginning the design of the collection at a high level of abstraction by a very simple model integrating general features only, then gradually increasing the features' less abstract models.

In the case of the cognitive hierarchy proposed here, a higher level in the cognitive hierarchy of agent may correspond to a lower level in the hierarchy of inheritance, in the spirit of Dennett's tower. We use both the possibility of specialising functions by redefining the corresponding methods, and the possibility of recursively using a "world" (i.e. a collection of agents in interaction) as an agent (Figure 1), in order to formalise the capability of the agent to model himself within his environment. Like with the decreasing abstraction methodology, the first simple model with a Reactive Agent enables us to understand the collective dynamics at the population level, and the more cognitive agents may introduce additional properties not reachable at a less "cognitive" level. From the point of view of Pattern, the relevant distinction is mainly between reactive / behavioural agents on the one hand, and epistemic agents on the other. The former requires only fairly simple computation rules (methods) for reactive agents, and more sophisticated rules for behavioural ones. The latter require at least two levels of hierarchy for a given agent: the agent himself, and the "mental representation" level, possibly (Figure 2).

Figure 2 : hierarchy of cognitive levels



Such patterns may help the modelling process, on the one hand by allowing the components to be reused, and on the other by assisting the modelling process at the conceptual level as well as the implementation level. A simple example taken from the results of statistical mechanics models to the modelling of economic systems with social interactions may highlight the connection.

An example of cognitive hierarchy of agents undergoing the same interaction principles: when do statistical mechanics principles apply to cognitive interactive agents?

In the Ising model of ferromagnetism, we may consider that the "agent" is a particle (the spin), that simply reacts to its environment (temperature, external field, interactions with other spins in the system), and that it possibly changes its state as a result of these forces. It is a "material" version of a reactive agent (RA). In models of discrete choice with externality, an “agent” is an economic agent (assumed to be human), who computes information (such as prices or neighbourhood choices) and who possibly changes his state as a result of these computations (to buy or not). Both models would inherit the methods from the more generic class “Agent”. But the specific methods and instances of agents are expected to differ between particles and customers. More specifically, an “economic” agent may be either a “reactive agent” or a more “cognitive” one, for instance behavioural or epistemic. The former behaviour is closer to that of a particle, for example, if we assume myopic behaviour. The latter may be more sophisticated, especially in the case of an epistemic agent who has the ability to model the behaviour of others in a strategic way

For Axtell (2000) there are three distinct uses of ACE: (1) classical simulations, (2) simulations that complement mathematical formulations, (3) simulations as substitutes for mathematical formulations. With the choice of Ising-related models of social interactions, we focus on the second case, when *ACE is used as a complement to classical mathematical approaches*. In particular, we are interested in identifying the case where the same mathematical models hold at all the levels of the cognitive hierarchy in the OOP representation, and the case where, if mathematical models exist, they are clearly different between one level and the other. It is clear that defining a cognitive hierarchy only makes sense in this last situation.

2 – FORMAL PRESENTATION OF BINARY MODELS

The Ising model of Statistical mechanics

The Ising model explains the physical properties of magnets. These are a consequence of interacting magnetic moments carried by the elementary particles that constitute the molecules of the solid. The magnetisation of a piece of condensed matter is a macroscopic observable, obtained by adding the contributions of the molecular moments, hereafter called spins in slight abuse of terminology. If each spin adopted any arbitrary orientation, the sum would be vanishingly small. This is indeed the case for most of the materials around us. In the presence of a magnetic field, magnetic moments exhibit a “preferred" orientation (the one that minimises the moments' magnetic energy), that is to be aligned parallel to the field. As a consequence, a macroscopic magnetic moment is observable. The Ising model enables us to understand how some materials, like iron, present permanent magnetisation in the absence of external magnetic fields

The model considers N spins s_i ($i = 1,...,N$) that may be oriented either up ($s_i = +1$) or down ($s_i = -1$). In the absence of spin interactions, the energy of an individual spin s_i in an external magnetic field h is: $E_i = -h.s_i$ A spin parallel to h has energy -h, while if it is anti-parallel, the energy is higher: +h. The total energy of a system of N non-interacting spins in an external magnetic field h is the sum of the individual energies:

$$(1) \qquad E = -\sum_{i=1}^{N} h.s_i$$

Now, consider a system where the spins interact with each other (each spin produces an effective microscopic magnetic field on the others). The field produced by spin k on spin i is proportional to the spin's own magnetic moment: $J_{ik}.s_k$. The constant of proportionality: J_{ik}, represents the strength of the interaction. The local field acting on spin i is the sum of the fields produced by its neighbours and the external field h,

$$(2) \qquad h_i = h + \sum_{k=1}^{N} J_{ik}.s_k$$

The spin's energy $E_i = h_i.s_i$ depends on the orientation of the neighbours through the exchange field $J_{ik}.s_k$. The total energy of a system of interacting Ising spins is:

$$(3) \qquad E = -\frac{1}{2}\left(\sum_{i=1}^{N} s_i \left(h + \sum_{k=1}^{N} J_{ik}.s_k\right)\right)$$

where the factor 1/2 is introduced to compensate for the double counting of each couple of spins (i,k) in the sum over i and k. Now the spins' orientations in the state of minimal energy, which are parallel to their local fields, cannot be as easily determined, due to the interactions. In the ferromagnetic case, in which the interactions are all positive ($J_{ik}>0$ for all i,k), in the fundamental state (the one of minimal energy) all the spins have the same orientation ; the corresponding magnetisation is maximal, equal to N. States of higher energy have a smaller magnetisation.

$$(4) \qquad P(S) = \frac{\exp(-\beta.E(S))}{\exp(+\beta.E(S)) + \exp(-\beta.E(S))}$$

Let us consider an observer interested in situations where the spins are not parallel, and on average the system's state has energy E. What are the states of the spins in that case? According to the principle of maximum entropy, the probability of a state S depends on its energy through the *Gibbs law*.

The basic model of binary choice with social influence

The basic economic model is drawn on a simplified version of Durlauf (1997) presented in Phan *et al.* 2004). A population of N agents are playing a game iteratively within their neighbourhood. Each agent has to make a binary choice s_i in the strategy set {-1, +1}. Agents are assumed to maximise an expected linear payoff

$$(5) \quad V(s_i) = \arg\max_{s_i} \left\{k + (h + \varepsilon_i).s_i + \hat{E}_i\left[S(s_i, s_{-i})\right]\right\}$$

This payoff embodies both a ”private” and a “social” component. In our very simple model, the private component includes the same common part for all agents: h and an idiosyncratic part: ε_i independent of the agents’ choice. If the distribution of ε_i over the agents has zero mean, h can be interpreted as the expected utility of agent i, in the case without social effects. The social component $S(s_i, s_{-i})$ measures the effect of the neighbourhood on the payoff of individuals. More formally, if ϑ_I is the subset of neighbours of the agent i, s_{-i} is the choice vector of these neighbours. $\hat{E}_i\left[S(s_i, s_{-i})\right]$ denotes an agent's i belief, about the effect of this neighbours’ choice on his payoff.

Assuming linear effect, let us denote J_{ik} the marginal effect of the agent k's choice upon the payoff of agent i, the cumulative social influence over the neighbourhood is :

$$(6) \qquad S(s_i, s_{-i}) \equiv s_i \sum_{k \in \vartheta_i} J_{ik}.s_k$$

Assuming that each agent i knows exactly all J_{ik}, expectations concerning only his neighbours' choices

$$(7) \qquad \hat{E}_i\left[S(s_i, s_{-i})\right] = s_i \sum_{k \in \vartheta_i} J_{ik}.\hat{E}_i\left[s_k\right]$$

The maximization of the utility can be rewritten as:

$$(8) \quad V(s_i) = \arg\max_{s_i} \left\{ k + s_i.\left(h + \varepsilon_i + \sum_{k \in \vartheta_i} J_{ik} \hat{E}_i\left[s_k\right]\right) \right\}$$

In the case where ε_i is a stochastic "trembling hand", the choice of agent i can be written as the conditional probability:

$$(9) \qquad \begin{aligned} &P\left(s_k = +1 \middle| \hat{z}_i\right) = P\left(-\varepsilon_i \le \hat{z}_i\right) \\ &\hat{z}_i \equiv h + \sum_{k \in \vartheta_i} J_{ik} \hat{E}_i\left[s_k\right] \end{aligned}$$

A formal analogy between this model and the Ising model of statistical mechanics appears if we assume *logistic distribution* with zero mean for the agents' random payoffs ε_i. Then, *the conditional probability of choice* can be rewritten as:

$$(10) \qquad P\left(s_i = \pm 1 \middle| \hat{z}_i\right) = \frac{\exp(\pm\beta.\hat{z}_i)}{\exp(+\beta.\hat{z}_i) + \exp(-\beta.\hat{z}_i)}$$

The related *mathematical conditional expectation* of agent i's choice, given his own expectations about the other agents' behaviours is:

$$(11) \quad E\left(s_i \middle| \hat{z}_i\right) = \frac{\exp(+\beta.\hat{z}_i) - \exp(-\beta.\hat{z}_i)}{\exp(+\beta.\hat{z}_i) + \exp(-\beta.\hat{z}_i)} = \tanh(\beta.\hat{z}_i)$$

If the individual stochastic terms are independent, the joint probability measure over the population follows a *Gibbs law*, like in the case of statistical mechanics. From a cognitive point of view, values in equations (8)-(11) all depend on the way each agent has subjective expectation $\hat{E}_i[]$. In order to focus on this question, we assume *homogeneous local interactions.* All neighbourhoods have the same size and all influence parameters J_{ik} are symmetric and equal, such as : $J_{ki} = J_{ik} = J_\vartheta = J/N_\vartheta$, where N_ϑ is the number of neighbours around an agent and J a positive parameter. For a given neighbour k taking in the neighbourhood ($k \in \vartheta$), the social influence is J_ϑ if the choices are the same, and $-J_\vartheta$ otherwise. As the cumulated social effect is the sum of individual effects over the neighbourhood, social influence depends on the *proportion* of customers in the neighbourhood, that is, individual influence is inversely proportional to the size of the neighbourhood.

Connection with game theory

Progress has been made recently in the theory of learning in games (Fudenberg, Levine, 1998) producing relevant results for learning in multi-agent systems (Vidal, 2003). It is possible to relate our model of binary choice with social influence to game theory in the framework of *population games* (Blume, 1997). Under our assumptions, all the agents have the same form of instrumental rationality, but may differ with respect to their cognitive approach on expectation formation $\hat{E}_i[]$. In the case without a trembling hand, ($\varepsilon_i = 0$), because each agent has only two possible strategies: ($s_i = +1$) and ($s_i = -1$), it is possible to represent his marginal payoff in each bilateral confrontation by a "normal form" matrix G_1 (player 1 in rows, player 2 in columns). The total payoff of these confrontations is equal to sum of all marginal payoffs from bilateral confrontation

G_1	$s_i = (+1)$	$s_i = (-1)$
$s_i = (+1)$	$k_\vartheta + h_\vartheta + J_\vartheta$	$k_\vartheta + h_\vartheta - J_\vartheta$
$s_i = (-1)$	$k_\vartheta - h_\vartheta - J_\vartheta$	$k_\vartheta - h_\vartheta + J_\vartheta$

where: $h_\vartheta \equiv h/N_\vartheta$, $k_\vartheta \equiv k/N_\vartheta$ and: $J_\vartheta \equiv J/N_\vartheta$. Let us consider the simple case: $k_\vartheta = 3$ $h_\vartheta = -1$; $J_\vartheta = +2$. (As usual, the first term is the payoff of player 1, the second the payoff of player 2).

G_1^*	$s_{i,k} = (+1)$	$s_{i,k} = (-1)$
$s_{i,k} = (+1)$	(4 , 4)	(0 , 2)
$s_{i,k} = (-1)$	(2 , 0)	(6 , 6)

Unsurprisingly, this bilateral game has two Nash equilibriums in pure strategy (+1) (+1) and (–1) (–1), when agents play the same strategy. In our *multilateral game*, each agent must choose the same single strategy with all the agents in his neighbourhood. That is, the total payoff depends on the proportion of opponents in the neighbourhood that play each of the available strategies. Let p be the proportion of (+1) strategists in the neighbourhood of an agent. For a given strategy, the average payoff of an agent playing a pure strategy in our multilateral game is equal to the mathematical expectation of this strategy:

$$(12) \qquad \begin{aligned} E(+1) &\equiv E\left(s_i = +1 \middle| p\right) = 4.p + 0.(1-p) \\ E(-1) &\equiv E\left(s_i = -1 \middle| p\right) = 2.p + 6.(1-p) \end{aligned}$$

For a "myopic" agent, who takes the previous strategy of his neighbours as given: $\hat{E}_i[s_k(t)] = s_k(t-1)$, the "best reply" is to play (+1), if :

$$(13) \qquad E\left(s_i = +1 \middle| p\right) > E\left(s_i = -1 \middle| p\right) \Leftrightarrow p > \frac{3}{4}$$

where the threshold p* = 3/4, obtained when E(–1) = E(+1) is the optimal proportion of pure strategy in the mixed strategies Nash equilibrium. More generally, condition (13) can be rewritten as :

$$(14) \qquad \begin{aligned} &E(s_i = +1 | p) > E(s_i = -1 | p) \Rightarrow \\ &p.(J_\vartheta + h_\vartheta) > (1-p).(J_\vartheta - h_\vartheta) \Rightarrow p > \frac{J_\vartheta - h_\vartheta}{2.J_\vartheta} \end{aligned}$$

Given condition (11) the game G1 is "best reply" equivalent to the following *pure co-ordination game* G2 (whose payoffs off the diagonal are zero).

G_2	s_i = (+1)	S_i = (–1)
s_i = (+1)	$J_9 + h_9$	0
s_i = (–1)	0	$J_9 - h_9$

The payoff of the two Nash equilibriums in this new matrix (+1)(+1): J + h, as well as: J – h for (–1)(–1), is proportional to the *cost of unilateral deviation from this equilibrium*, Thus, following Harsanyi, Selten (1988), the equilibrium with highest *cost of unilateral deviation (ie* payoff) is said to be "*risk-dominant*", and the other is said to be "*risk-dominated*". The *risk-dominance* criterion means that it costs less to deviate from a *risk-dominated* Nash equilibrium than from a *risk-dominant* one. As a result, with instrumentally-rational players, the risk dominant position is less risky, because it is costly to deviate unilaterally. In our example, with h = –1 et J = 2, agents' preferences are closer to s_i = (–1) and the related risk-dominant Nash equilibrium is (–1) (–1)

G_2^*	s_2= (+1)	s_2 = (–1)
s_1 = (+1)	(1, 1)	(0, 0)
s_1 = (–1)	(0, 0)	(3, 3)

This class of game with " best reply equivalence" is called a (weighted) *potential games* (Monderer, Shapley, 1996). One interest of a potential game is that it defines a class of games with the same best reply behaviour, *ie*. the same equilibrium, and, in certain cases, the same dynamic (when the dynamic is related to the binary response, not to the payoff level). In our case, we have a special interest in the so-called "stag-hunt" game, and related social consequences. By adding +4 in both values in the first column of matrix G_2^*, we have an example of the "stag-hunt" game, *best reply equivalent* to both G_2^* and G_1^*.

G_2^*	s_2= (+1)	s_2 = (–1)
s_1 = (+1)	(5, 5)	(0, 4)
s_1 = (–1)	(4, 0)	(3, 3)

In other words, all models of binary choice with social influence such as $h_9 = - J_9 / 2$ (in particular for all k) are "best reply equivalent" to a "stag-hunt" game, both in the Nash equilibrium properties and for the class of dynamics based on the binary best response and independent of the payoff level. In such a game, the *risk dominant* equilibrium (–1)(–1) has a lower payoff but a higher cost of deviation. It is said to be "*Pareto dominated*" by the second (+1)(+1), which is said to be "*Pareto dominant*" (higher payoff), but risk dominated (lower cost of deviation). As a result, the socially efficient equilibrium, with high payoff, may not be attainable for a large class of dynamic processes.

For instance, Blume (1993) first introduced a model where ε_i is a stochastic "trembling hand" in population game theory with explicit reference to statistical mechanics, following a logistic distribution (called the *stochastic best response dynamics* – or the *log-linear response with parameter β*, by Young, 1998). This model based on *myopic best reply* are coherent with standard results in statistical mechanics, because the model is basically the same when $\hat{E}_i[s_k(t)] = s_k(t\text{-}1)$. In particular, for a size of the population sufficiently large and for a sufficiently large value of β (small value of variance of ε) the invariant distribution of asymptotic states puts almost all the probability within a neighbourhood of the risk-dominant equilibrium. This situation, similar to the one which prevails for a linear stochastic "trembling hand", is said to be *stochastically stable* (Young 1998) From Statistical Mechanics, the same result holds for very small values of of β (great value of variance of ε). Both small noise and large noise generate uniqueness in stochastic best response dynamics, as simulations clearly confirm.

To summarise, on the one hand, with behavioural myopic agents (low level of cognition), we have exactly the same behaviour as with quasi-reactive particles in statistical mechanics. As a result, the socially Pareto superior equilibrium may be not attainable. On the other hand, with eductive agents with "rational expectations" all expectations turn to the same mathematical expectations and the model is equivalent to the *mean field approximation* of statistical mechanics (Brock, Durlauf 2001, Phan *et al*. 2004). Our challenge is to investigate the case with real learning, and our investigation into cognitive hierarchy will take these two special cases as references.

3 – THE EFFECTIVE COGNITIVE HIERARCHY: SOME PRELIMINARY RESULTS

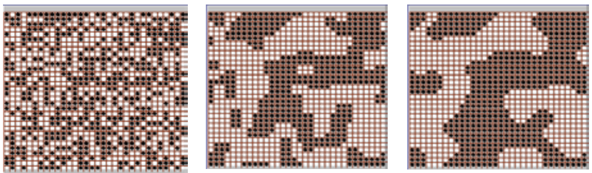
For this first investigation into cognitive hierarchy, additional assumptions are able to simplify the comparison between simulations and analytical issues. It is interesting to compare the case described with a "trembling hand" (stochastic ε_i,) and the case where agents have a private idiosyncratic component ε_i, randomly distributed over agents at the beginning of the period under consideration, but which remains fixed afterwards (dynamic with synchronous updating at each step within the period). Following Phan *et al*. (2004), it is possible to underline a formal equivalence between this economic model and the *Random Field Ising Model* (RFIM), at zero temperature of statistical mechanics. Moreover, for physicists, this latter model with fixed heterogeneity belongs to the class of *quenched disorder*, while the case with a "trembling hand" belongs to the class of *annealed disorder* (dynamic with stochastic ε_i and asynchronous updating at each step). Future investigations will take into account both forms of heterogeneity.

Reactive rule: myopic best reply

Simulations focus on the question of selection between strategies. Assuming initial distribution of the population without externality effect (J=0), and without bias between s_1 and s_2 (h = 0), what kind of convergence toward equilibrium is it possible to observe for J>0 ? Each configuration puts the question of relevance of (1) simulation and (2) multi-agent system. All simulations are on a torus with N= 32x32 = 1024 agents in three cases : neighbour Von Neuman (4) Neighbour Moore (8) and Neighbour "World" (full connectivity).

The Game-theoretic approach with myopic best reply follows the same model as statistical mechanics. Simulation processes and results are exactly the same in both cases. For *quenched disorder,* with neighbour "World" (complete connectivity), simulations converge toward one equilibrium or an other, given the initial distribution of the population (initial distribution remains stationary if p is exactly 50%). With neighbours Von Neuman (4) and Neighbour Moore (8), simulations converge towards stable clusters around 50 % of the population, (with standard deviation of 4,75% for VonNeuman and 6,82% for Moore). Final configurations are between 30% and 70% of the population with Neighbour Moore. Sometimes local cycling configurations appear along the frontiers (more frequently in Von Neuman)

Figure 1 : clusters with myopic best reply on a torus



initial distribution | NeighbourVonNeuman | NeighbourMoore

With myopic best reply, the convergence is very fast, but agents do not learn anything. With complete connectivity, and finite population, the final Nash equilibrium is predictable by the initial bias. In the special case where there is exactly the same number of (+1) and (−1) players: the initial distribution is an equilibrium. In the case of *annealed disorder*, (trembling hand), with a limited set of simulations, the dynamic converges towards a risk dominant equilibrium even with a local neighbourhood, as local clusters gradually collapse.

Behavioural rule: exploration exploitation via CPR

According with the behavioural level of cognition, the Skinnerian agent does not have to model the behaviour of his neighbours, but may experiment through a trial-and-error process. An example of behavioural rule is the "*cumulative proportional reinforcement rule*" (CPR), first suggested by Bush, Mosteller (1995), and recently studied in particular by Arthur (1993) and Laslier *et al*. (2001). With this rule, the probability of experimenting a given solution decreases if the relative part of payoff linked with this solution decreases (self-reinforcement effect). That is, at the initial period, the probability of playing each strategy is equal, except if the agent has some prior information of the corresponding payoff. The learning process increases smoothly the probability of choosing the most successful strategy in the past. Formally, the probability of choosing strategy $h \in \{-,+\}$ at time t is proportional to the cumulative utility of this strategy :

$$p_h(t) = \frac{\sum_{\tau=0}^{t} \delta_h(\tau).U_h(\tau) + U_h(0)}{\sum_{\sigma \in \{+,-\}} \left(\sum_{\tau=0}^{t} \delta_\sigma(\tau).U_\sigma(\tau) + U_\sigma(0) \right)} \qquad (17)$$

where $\delta_\sigma(t)$ is a Kronecker function such that: $\delta_\sigma(t) = 1$ if (+1) is played at time t, and 0 otherwise. $U_\sigma(0)$ is taken as the total initial payoff of both strategies. In the present case study, because of the non-stationarity of the strategies, CPR alone is not relevant and introduces only perturbations along the initial position, when myopic best reply allows fast convergence. In order to maintain a small degree of exploration, we use a mixed myopic best-reply / CPR rule, as follows. The CPR rule only applies when the alternative strategy has a greater payoff than the current strategy. In the case where the current strategy has the greater payoff, CPR applies randomly with probability q. When q = 1, the rule is fully CPR. With q= 0, the CPR rule applies only when the alternative strategy has a greater payoff than the current one. With quenched disorder, the smaller q is, the faster is the convergence. The mixed rule allows only to break the indeterminacy in the case where there is exactly the same number of (+1) and (−1) players. Because it is a self-reinforcement rule, the process evolves like a Polya Urn (cf. Laslier *et al*. 2001), and the first random explorations are critical for the convergence towards one Nash equilibrium or another.

Epistemic rule

In our multi-agents framework, this epistemic level can be formalised by an agent handling a "world" of agents in his "head". More specifically, each agent *compute*() method uses the services of a specific instantiation of a "World" class, which contains Agents having a kind of oversimplified "mirror" of their neighbourhood (in their "head"). The resulting process of learning is *epistemic*, because each agent has a model of himself in interaction with the other agents based on prior belief, and updates his beliefs, according to observed facts. This process of learning is bounded *by construction*, because the length of recall is limited to some recent periods. In some cases, the size of the world-in-the-head of the agent is itself bounded by the size of observed neighbours (4 or 8). Additional assumptions can be justified in this preliminary work by possible non-stationarity of the strategies played on the network, due to the indirect effect of externality between agents, and "domino" effects (Phan *et al.* 2003). In another implementation of epistemic agents (Dessalles, Phan 2004) agents have a two level synthetic model of recognition of mixed strategy in the population and the detection of effect of signalling on the behaviour of the others

CONCLUSION

In the context of the model used in this paper, neither the reactive rule nor the behavioural rule really needs a multi-agent framework (except for easy-to-handle, user friendly interfaces) but need simulation both for investigation and to produce results, for instance, in the case where agents interact in a structured network. In such a case, the agent-based framework in OOP with increasing cognitive hierarchy is very powerful in making the code clear and brief, by reusing components from a more abstract level. In a more sophisticated model of cognitive agents, with more sophisticated expectation, the agent-based model would be more and more useful. This is also the case when other dimensions will be embedded, such as communication or socially situated rationality rules.

From the economic point of view, ACE embodies the two sub-perspectives of cognitive economics: the epistemic one and the evolutionary one (Walliser, 2004}. The integration of both dimensions in the same framework is a real challenge. Currently, the conceptual and formal integration of the two dimensions within a significant and coherent analytical framework need more development. The strategy suggested here is to keep the connection between these two approaches and to use ACE as a complement of the analytical one, to investigate complex dynamics linked with both social interactions and belief revisions. However, the integration of these two dimensions would seem to be a major challenge for the coming years.

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AUTHOR BIOGRAPHY

DENIS PHAN is a Post & Telecommunications administrator and Senior Lecturer in Economics at ENST de Bretagne (ENST). His past research interests are in the economics of telecommunications, the software industry and regulation. He has published papers and various books in this field. Since 2000 his main research interests have been cognitive and computational economics. Together with Antoine Beugnard, a computer scientist, he built *Moduleco*, a Multi-agent framework, in order to simulate markets, social networks and other complex adaptive system phenomena, in co-operation with physicists.

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